Time-Domain Non-Linear Noise Analysis of FET Oscillators

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Abstract — **A novel approach to the noise analysis of FET oscillators is presented. A highly accurate simulation environment for oscillator analysis is constructed, based on full time-domain transient analysis using an equivalent circuit type device model. Internal noise sources are introduced as random signals in the time-domain, with a frequency spectrum matched to the physical spectrum of the appropriate noise mechanism. The channel thermal noise as a function of the periodically varying channel resistance, is modeled in the time-domain, as is the thermal resistance at the gate and at terminations. Flicker noise within the device is also considered. The effects of these noise sources near the oscillation frequency can be seen in the phase noise sidebands, with the flicker noise dominant closer to the oscillation frequency. Presented results show phase noise spectra in good qualitative and quantitative agreement with experimental results for typical FET oscillators.**

I. Introduction

Non-linear noise arises when internal noise sources are converted and mixed to produce AM, FM and PM modulations representing unwanted signal energy in a band of interest. This noise is of particular concern in modern RF transceiver design because it decreases the selectivity of the receivers and increases the bandwidth of the transmitted signals. Since the demand on frequency channels for mobile communication applications has greatly increased in the last few years, characterizing how noise affects oscillators is vital to future technological advances in this area. Therefore, it is crucial to the design of transceivers to have both accurate noise models for the devices from which they are constructed, and accurate methods of noise performance analysis. Phase noise, which refers to the short-term random fluctuation in the frequency of an oscillator signal, is a phenomenon that has yet to be satisfactorily characterized.

Much of the earlier work carried out in this area applies either linear time-invariant (LTI) [1] or linear time-varying (LTV) [2]-[3] assumptions, which neglect the fundamental issues involved, and are also incapable of modeling the noise spectrum close to the carrier. Most of these types of analysis treat the noise as a first-order linear perturbation to the steady-state solution of the system, which is generally determined by the harmonic balance (HB) technique. The method of phase noise analysis using the HB technique [4] is only practical for weakly non-linear circuits with low noise levels, as it requires large amounts of memory and computational time for highly non-linear circuits.

However, in recent years much productive work has been carried out in an attempt to develop new methods of characterizing noise under non-linear conditions, and its effect on system performance and output [5]-[10]. These methods can be classed as time-, frequency-, or mixed-domain methods. Conventional Newton-iteration-based HB algorithms have been replaced by more efficient conversion algorithms [5], [8]. Reference [5] adopts the inexact-Newton HB technique to analyse large-signal noise effects of a power amplifier with white noise sources in the frequency-domain. Some inventive work in the time-domain is presented in [6], which applies Floquet's theory to calculate a periodic sensitivity function to the noise perturbations. Building on this approach, [10] demonstrates how these sensitivity functions can be calculated through the HB method. However, recent work by [11] has questioned the representation of phase noise in [6] as a single scalar quantity. Mixed-domain methods such as the envelope transient method [8]-[9] have been shown to present considerable reductions in computational effort over their purely time- or frequency-domain counterparts.

In this work a dedicated oscillator simulation tool was developed in a C-code, which allows the inclusion of intrinsic noise sources in an accessible way. The time-domain method presented here can accurately simulate highly non-linear circuits, which in this case is based on the COBRA drain current equivalent circuit FET model [12], while including flicker noise and channel thermal. noise as the main contributors to the phase noise. Thermal noise generated from parasitic resistances is also considered, and each noise source is treated as uncorrelated, since each is generated from an independent physical source. The thermal noise is generated by a summation of sinusoids with pseudo-random phase at discrete frequencies over the examined bandwidth. Similar methods of noise generation are presented in [5], [7], which involve Monte Carlo analyses. The amplitude at any discrete frequency is given by Planck's black body radiation law. Therefore, the thermal noise is appropriately shaped in the frequency spectrum. A similar method is adopted for producing the flicker noise source, with a typical empirical factor employed to simulate the correct magnitude.

To the author's knowledge, this is the first time that phase noise in noisy oscillators has been examined exclusively in a transient analysis simulation with noise sources modeled as true signals in time. Unlike HB noise-analysis methods, and some time-domain methods, there is *no* small-signal noise assumption, i.e. the deterministic signal and the noise signals are *not* treated separately. A linear model is *not* used for noise analysis. This method therefore incorporates the full non-linearities of the noisy oscillator. Since the oscillation implies large signal periodically varying power levels, the inherent transistor non-linearities give rise to out of band noise conversion into the band of interest about the oscillation frequency, which can be accurately observed via the FFT. The proposed method is also more computationally efficient than a Monte Carlo time-domain noise analysis, where many transient runs of the noisy oscillator are required.

II. Noise model formulation

The noise sources within the system are each represented as a set of uniformly spaced sinusoids with statistically independent pseudo-random phases. Any noise source is a sum of N_f sinusoids:

$$n(t)=\sum_{i=1}^{N_f} A_i(t).\cos(2\pi f_i t+\varphi_i) \tag{1}$$

where $A_i(t)$ is the noise magnitude at a particular frequency, f_i is the frequency of each sinusoid and φ_i is the pseudo-random phase associated with each sinusoid. The pseudo-random phases have a uniform probability distribution in [0 2π]. The sinusoid frequencies are considered up to an appropriate maximum frequency, and are also simulated to a certain stop-time, which is determined by the required resolution. If the bandwidth, Δf, between adjacent discrete signals is infinitesimal, the noise power in that interval may be represented by a single sinusoid of the appropriate magnitude. Therefore, as the bandwidth between the sinusoids decreases, the modeled noise spectrum becomes increasingly similar to a continuous real noise spectrum. The magnitudes of the sinusoids at each frequency are matched to the spectrum characteristics of that particular noise source (see Fig. 1). Since noise source magnitudes are typically represented as RMS values, and the nature of the noise models presented here constitute a sum of sinusoids, a factor of $\sqrt{2}$ must be multiplied by the RMS values to give the correct amplitude, $A_i(t)$.

The noise power over a certain bandwidth for a sufficiently large number of these sinusoids approaches a mean value to an acceptable confidence level [5]. The frequency resolution of the simulation is limited by both the number of points in the FFT taken from the time-domain steady state sample, and the sampling rate. The resolution for an equal number of points can be improved by sampling at a lower frequency than the inverse of the time-step used in the simulation, i.e. by Fourier transforming only every n^{th} sample in the transient simulation.

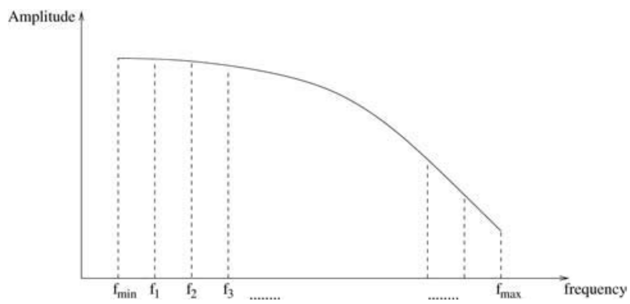

Fig. 1. An example of a physical noise spectrum

III. Noise Sources

Thermal noise, produced by small, random voltage fluctuations in resistive elements, has a zero average value, but a nonzero RMS value given by Planck's black body radiation law,

$$V_n=\sqrt{\frac{4hfBR}{e^{hf/kT}-1}} \tag{2}$$

where h = 6.626e^{-34} J-sec is Planck's constant, k=1.38e^{-23} J/°K is Boltzmann's constant, T is the temperature in degrees Kelvin (K), B is the bandwidth of the system in Hz, f is the centre frequency of the bandwidth in Hz, and R is the resistance in Ω. The amplitude of each sinusoid is determined from (2), where B becomes the bandwidth between adjacent sinusoids, with f becoming the sinusoid frequency.

The thermal noise exhibited by the resistive drain-source channel is modeled in a similar way by a noise current, Id_n, whose amplitude is dependent upon the periodically varying transconductance, g_m:

$$Id_n=\sqrt{\frac{4hfBg_m\gamma}{e^{hf/kT}-1}} \tag{3}$$

where γ typically has a value of 2/3. This formula is used in many simulators [13] in both the saturation and linear regions, which can lead to erroneous results in the linear region and around pinch-off. For instance, (3) erroneously predicts zero thermal noise when V_{ds}=0 (since then g_m=0), which is physically impossible. However, for our application, which will operate in the saturation region, (3) suffices.

Flicker noise is constructed in a similar way. The exact origin of flicker noise has been frequently debated over the years [14], but in FETs it is generally attributed to the random fluctuation of the number of charge carriers in the channel, due to trapping effects associated with crystal defects. Flicker noise has an amplitude distribution that is inversely proportional to frequency, and is generally modeled by way of empirical constants. The RMS value of the flicker noise in the simulation is modeled by the formula:

$$I_f = \sqrt{K \frac{I_D^{\alpha}}{f^{\beta}} B} \tag{4}$$

where I_D is the DC drain current in the FET, α is an empirical factor with typical values between 0.5 and 2, K is the flicker noise coefficient of the device, typically of the order of 10^{-12} for FETs, and B is the bandwidth of the system in Hz. If β=1, the noise spectral density has a 1/f dependence near baseband. The magnitude of the flicker noise coefficient has been empirically chosen to be in agreement with typical GaAsFET values.

IV. Implementation

A working oscillator circuit has been designed and then implemented in the C-language simulator developed here in order to test the capability of the proposed method for oscillator phase noise analysis. The system of non-linear differential equations describing the oscillator was solved by numerical integration at each time-step with the fourth-order Adams-Moulton predictor-corrector method. An accurate simulator tool with a numerical noise floor of the order of –350dBm was developed by these means, which allowed for the examination of the individual effects of each noise source on the oscillation phase noise. Both the Curtice-Ettenberg GaAsFET drain current model and the COBRA GaAsFET drain current model were implemented in a separate amplifier simulation, with the latter proving more accurate in comparison to typically measured results, especially around pinch-off. For this reason, the COBRA model was used in the oscillator simulation.

Five of the main noise sources were examined – the varying transistor channel thermal noise, gate resistance noise, flicker noise in the channel, and the two terminating resistance thermal noises. All of these noise sources originate from independent physical mechanisms and are therefore uncorrelated in the simulations. This is achieved by using a different seed for each noise source in the random number generator that produces the pseudo-random phases. The small-signal noise capabilities of the model were initially tested in a separate GaAsFET amplifier simulator with both drain current models, and it proved accurate in terms of noise figure comparisons to ADS and to typical GaAsFET noise characteristics.

V. Results

Fig.2 shows the intrinsic device used in the oscillator, together with the external circuit environment and the appropriate noise sources. The simulated oscillation was examined at the drain node, and transformed via the FFT in Matlab to the frequency-domain. Fig. 3 shows the simulated phase noise generated about the fundamental oscillation frequency of 2.145 GHz, when only flicker noise is considered. Fig. 4 shows the phase noise when all simulated noise sources are included. These results show that flicker noise is dominant closer to the oscillation frequency as would be expected. This method of analysis has the capability to accurately determine the near-to- carrier noise, as the only factors limiting the proximity to the carrier are the length of the time sample that is Fourier transformed, and the integration time-step. The development of accurate noise models can therefore be pursued by comparison with system measurements. The results shown in Figs. 3 and 4 are considered very encouraging. In particular, the magnitude of the phase noise in Fig. 4 is typical of a GaAsFET oscillator, and phase noise levels reported for similar devices [15] are in good agreement with those simulated here.

As can be seen in Fig. 2, a simple LC resonator has been used as the feedback in the oscillator. Figs. 3 and 4 show the phase noise for resonators of equal Q-factor, with only flicker noise included for the phase noise in Fig. 3. In Figs. 4 and 5, all noise sources are considered, with a higher Q-factor resonator used in generating the phase noise in Fig. 5. A decrease in the phase noise follows, as would be expected. Fig. 4 also shows the effect of using different seeds for the random phases in the noise models. This produces various phase noise levels that are similar in magnitude to an average value at each discrete frequency, thereby justifying the proposed method.

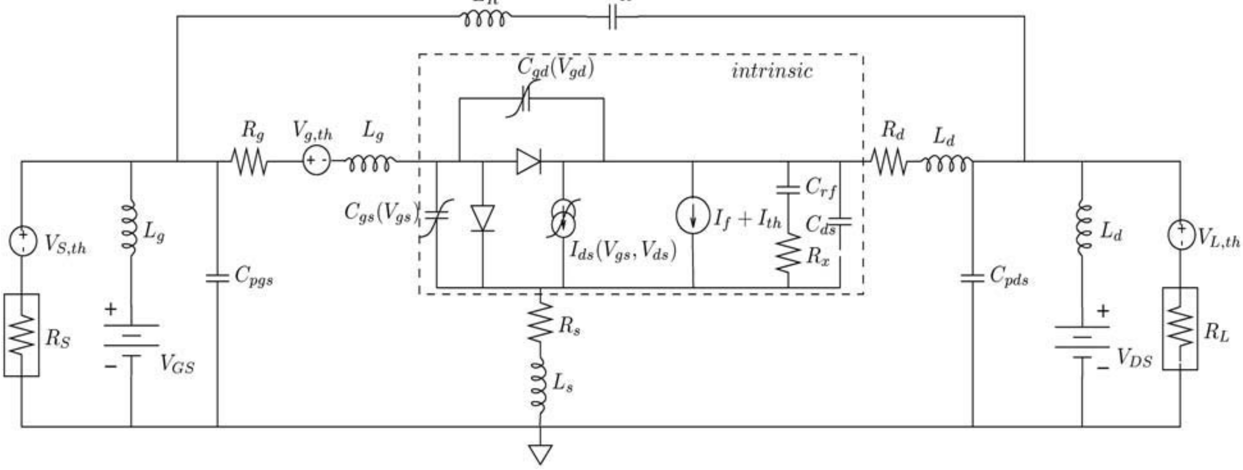

Fig. 2. Large signal noise equivalent circuit with all noise sources included: flicker and channel thermal noise are represented as a single noise source.

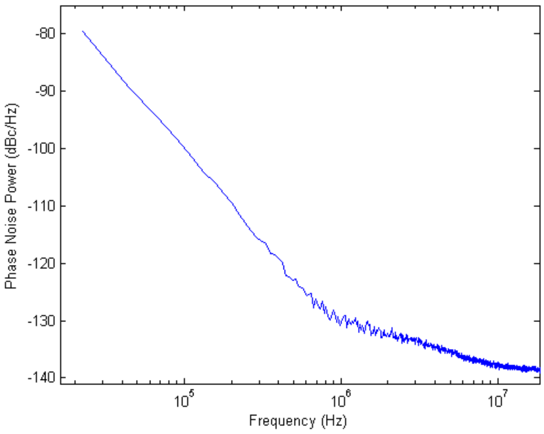

Fig. 3. Phase noise with only flicker noise considered. Note the different fall-off rates of the simulated phase noise.

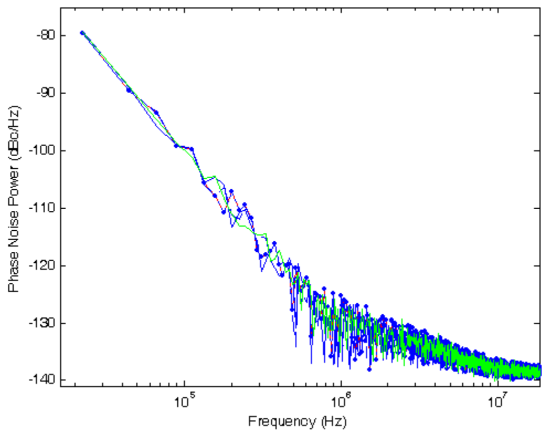

Fig. 4. Phase noise, with all noise sources considered and different seeds.

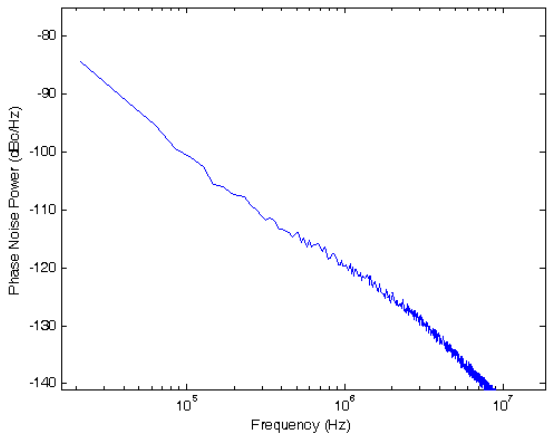

Fig. 5. Phase noise, with all noise sources considered and a higher Q-factor

VI. CONCLUSION

A novel time-domain method of noise modeling and analysis in FET oscillators has been presented. Phase noise levels simulated are consistent with those typically found in GaAsFET oscillators. Using the simulation approach demonstrated here, the elements that most strongly effect phase noise can be studied and separately identified, and designers can be enabled to develop circuit designs that better optimize phase noise performance in FET-based oscillators.

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REFERENCES

[1] D. B. Leeson, “A simple model of feedback oscillator noise spectrum,” *Proc. IEEE*, vol. 54, no. 2, pp.329, Feb. 1966.

[2] V. Rizzoli, A. Costanzo, F. Mastri, and C. Cecchetti, “Harmonic-balance optimization of microwave oscillators for electrical performance, steady-state stability, and near-carrier phase noise,” *IEEE MTT-S Int. Microwave Symposium Digest*, May 1994.

[3] M. Okumura and H. Tanimoto, ”A time-domain method for numerical noise analysis for oscillators*,” Proc. ASP-DAC*, Jan. 1997.

[4] V.Rizzoli, F. Mastri, and D. Masotti, “General noise analysis of nonlinear microwave circuits by the piecewise harmonic-balance technique,” *IEEE Transactions on Microwave Theory and Techniques*, vol. 42, no. 5, pp. 807-819, May 1994.

[5] V. Rizzoli, C. Cecchetti, and F. Mastri, “A rigorous frequency-domain approach to large-signal noise in nonlinear microwave circuits,” *IEEE Microwave and Guided Wave Letters*, vol. 8, no. 6, June 1998.

[6] A. Demir, “Phase noise in oscillators: DAE’s and coloured noise sources,” *Proc. IEEE/ACM Int. Computer-Aided Design Conf.*, pp. 170-177, Nov. 1998.

[7] P. Bolcato and R. Poujois, “A new approach for noise simulation in transient analysis,” *Proceedings of IEEE Int. Symp. On Circuits and Systems*, vol. 2, pp. 887-890, May 1992.

[8] S. Sancho and A. Suárez, “Efficient analysis of phase-locked loops through a novel time-frequency approach, based on two envelope transient formulations,” *IEEE MTT-S Int. Microwave Symp. Digest*, vol. 3, pp. 2153-2156, June 2003.

[9] E. de Cos, S. Sancho, and A. Suárez, “Noise analysis of modulated self-oscillating mixers,” *IEEE MTT-S Int. Microwave Symp. Digest*, vol. 3, pp. 1875-1878, June 2004.

[10] A. Suárez, S. Sancho, S. Ver Hoeye, and J. Portilla, “Analytical comparison between time- and frequency-domain techniques for phase-noise analysis,” *IEEE Transactions on Microwave Theory and Techniques*, vol. 50, no. 10, October 2002.

[11] J.C. Nallatamby, P. Bolcato, M. Prigent, R. Larcheveque, and J. Obregon, “Phase noise: whether identical at all nodes in free-running oscillator circuit,” *IEE Elec. Letters*, vol. 14, no. 8, pp. 460-461, April 2005.

[12] Cojocaru V.I. and Brazil T.J., “A scalable general purpose model for microwave FET’s including DC/AC dispersion effects”, *IEEE Transactions on Microwave Theory and Techniques*, vol. MTT-45, pp.2248-2245, December 1997.

[13] L. Nagel: ”SPICE2: A computer program to simulate semiconductor circuits,” *Electronics Research Laboratory Report*. No. ERL-M520, Univ. of Cal., Berkeley, 1975.

[14] F.X. Kaertner. “Analysis of white and f^{α} noise in oscillators”, *International Journal of Circuit Theory and Applications*, vol. 18, pp. 485-519, 1990.

[15] Chang-Li Chen, Xing-Nan Hong and Bao-Xin Gao, “Simulation of the accurate near-carrier phase noise in microwave MESFET oscillators”, *IEEE MTT-S IMS Digest,* vol. 3, pp. 1519-1522, May 1995.