Accurate and Efficient Incorporation of Frequency-Domain Data within Linear and Non-linear Time-Domain Transient Simulation

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***Abstract* — An effective method is presented for the representation in (non-uniform) discrete-time of all the S-parameters of an arbitrary linear, time-invariant network specified up to some given maximum frequency. The proposed representation is shown to be highly accurate, compact, stable and derivable using straightforward numerical procedures. Furthermore it is in a form that can be readily incorporated within linear or non-linear time-domain transient simulation, producing results with very high efficiency. Several examples are given providing independent verification of the technique.**

***Index Terms* — Transient simulation, convolution, S-parameters, non-linear**

I. INTRODUCTION

A fundamental and commonly encountered problem in high-frequency simulation is that of incorporating known frequency-domain data (e.g. S-parameters) within a time-domain simulation. Many approaches to this problem have been proposed over the years, most commonly involving some kind of rational function (e.g. Padé) approximation to the given data followed by recursive convolution within a time-domain simulation [1]-[4]. The great majority of this work has been directed to the special case of lossy multiple-conductor transmission lines. These approaches suffer from a number of serious drawbacks, however, including difficulties with reliable extraction of the polynomial coefficients together with general problems associated with achieving good accuracy under general conditions while also preserving stability and causality.

We describe the successful development of an alternative approach to this problem, with the objective of representing in the time-domain all of the S-parameters of a completely arbitrary N-port linear, time-invariant network that are available up to some maximum frequency of interest f_m. It will be shown how each complex-valued S-parameter can be represented with extremely high frequency-domain accuracy through a particular formulation based on a non-uniformly-spaced, compact, finite sequence of discrete-time, real-valued impulse response weights. The resulting representation is easily obtained and is ideally suited to implementation within a linear or non-linear time-domain transient simulator. Several examples are given which validate the accuracy of the frequency-domain representation as well as the accuracy of both transient and steady-state responses in linear and non-linear time-domain simulation. The results also demonstrate exceptional simulation speed, stability and accuracy.

II. NON-UNIFORM DISCRETE-TIME REPRESENTATION OF FREQUENCY-DOMAIN DATA

In principle the task at issue can be performed using the Fourier Transform/FFT followed by a convolution between the resulting impulse response and the input signal. In practice, this is far from straightforward except in very special cases. A basic problem is that the given data is known only up to the maximum frequency f_m and fairly obvious strategies such as using a 'window' function to kill off the given function outside the known band can create serious inaccuracies. Periodic extension appears attractive but seems very difficult to manage in general cases for both reflection and transmission parameters.

Consider now a given (Hermitean) S-parameter function $S_{ij}(f)$ = $(S^r_{ij}(f)+j.S^i_{ij}(f))$, either reflection or transmission in kind, referred to G_o and specified at (N+1) equally-spaced tabulated values of f ε [0, f_m] with interval Δf. A direct periodic extension of the given data beyond the known range [$-f_m$, f_m] will, in general, introduce major complex-valued discontinuities at each boundary frequency. If it were then attempted to convert a function of this kind into the time-domain, the resulting impulse response would be of very long duration and/or would have very poor interpolation properties back into the frequency-domain between the original data points. Consider now the new function:

$$F_{ij}(f) = \left[S_{ij}(f) - K\right] \cdot e^{-j\cdot\varpi\cdot\tau} \qquad (1)$$

where K and τ are real numbers, and τ ε [0, (1/2.f_m)]. It is proposed to choose these numbers to satisfy the following two simultaneous conditions:

(1) Im{$F_{ij}(f_m)$} = 0.0. For a Hermitean S_{ij} this is sufficient to avoid a discontinuity in F_{ij} at f=f_m. F_{ij} is thus continuous and periodic in a complex-valued sense and may be represented efficiently by a discrete time sequence of impulse response weights separated by Δtir=(1/2.f_m);
(2) The impulse response weight calculated as a result of condition (1) at time = 0 is forced to be exactly zero.

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The resulting time-domain representation of $F_{ij}(f)$ may resemble that shown in Fig. 1. However, our original objective was to obtain a discrete-time representation of the scattering parameter $S_{ij}(f)$. This is achieved as follows:

(a) Remove the additional phase shift introduced by the exponential term in Eq. (1). This corresponds to shifting the entire response in Fig. 1 by an amount τ to the left, i.e. in the direction of negative time, and explains the need for condition (2), since moving the first, zero-valued weight into the negative time region does not then led to a violation of causality.

(b) If K is non-zero, introduce an additional impulse at time 0 of value (K. Δtir). The discrete-time representation of $S_{ij}(f)$ is therefore as shown in Fig.2., and is potentially non-uniform adjacent to the origin, but uniform otherwise. Typically of the order of 50-100 weights are sufficient to represent completely even quite complex frequency-domain behavior.

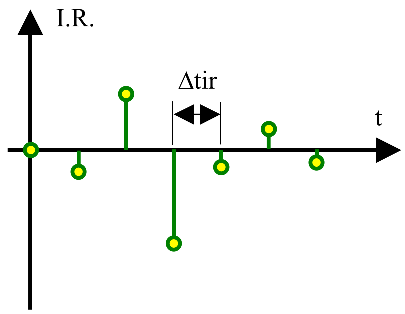

Fig. 1. Time-domain representation of periodic extension of $F_{ij}(f)$ in Eq. (1), satisfying both conditions (1) and (2)

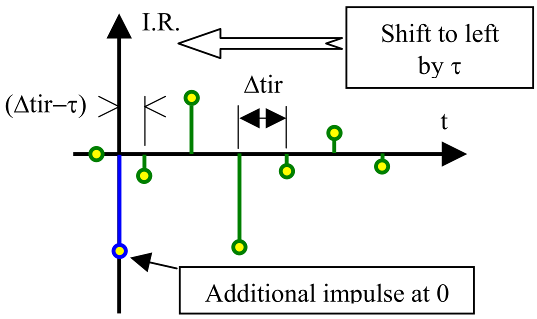

Fig. 2. Time-domain representation of Scattering Parameter $S_{ij}(f)$ derived from response in Fig. 1.

It can be shown that the procedure required to satisfy simultaneously conditions (1) and (2) reduces to determining a value of τ in the range $[0, (1/2.f_m)]$ such that:

$$\Delta f \cdot \left[\sum_{i=1}^{N-1} 2\left\{ S_{ij}^{r}(i\Delta f) - K \right\} \cdot \cos(\varpi_m \cdot \tau) + S_{ij}^{i}(i\Delta f) \cdot \sin(\varpi_m \cdot \tau) \right]$$
$$+ \left[S_{ij}^{r}(0) - K \right] + \left[S_{ij}^{r}(f_m) - K \right] \cdot \cos(\varpi_m \cdot \tau) +$$
$$S_{ij}^{i}(f_m) \cdot \sin(\varpi_m \cdot \tau) = 0$$

where

$$K = S_{ij}^{r}(f_m) - \left(S_{ij}^{i}(f_m) / \tan(\varpi_m \cdot \tau) \right)$$

This is a straightforward numerical exercise in finding a bracketed root of an algebraic non-linearity.

As an example of the application of this method consider an arbitrary distributed/lumped linear TPN as depicted in Fig. 3.

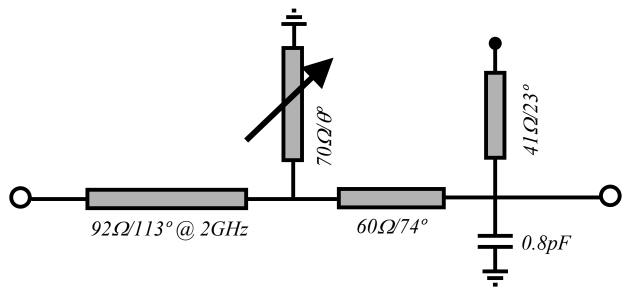

Fig. 3. Lossy distributed/ lumped test network. Stub length θ varied over 3 values: 11°; 54°; 97°.

The transmission lines are highly non-commensurate and further exhibit significant loss and dispersion. To provide a more robust test of the procedure, one of the transmission line lengths is tuned over three separate electrical lengths producing very different frequency-domain responses. Then, starting with just 32 frequency-domain samples in the range between DC and 12GHz, the procedure in section II was used to compute the time-domain representation for each S-parameter. The results in Fig. 4 show the frequency-domain analyses of the original circuit (for the three lengths) using a high resolution in the frequency-domain, compared to the predictions of the kind of time-domain representation shown in Fig. 2 for S_{11} and S_{21} in terms of both magnitude and phase (S_{22} is similar).

The time-domain representation for S_{11} is also given in Fig. 5 showing a detail of the region near time zero to emphasize the non-uniformity in time of the IR weights in this region for the 3 cases. It is important to note that these are fully interpolated results: all of the time-domain data records have been augmented with a significant number of zeroes to provide greatly enhanced frequency-domain resolution for the purpose of making these comparisons.

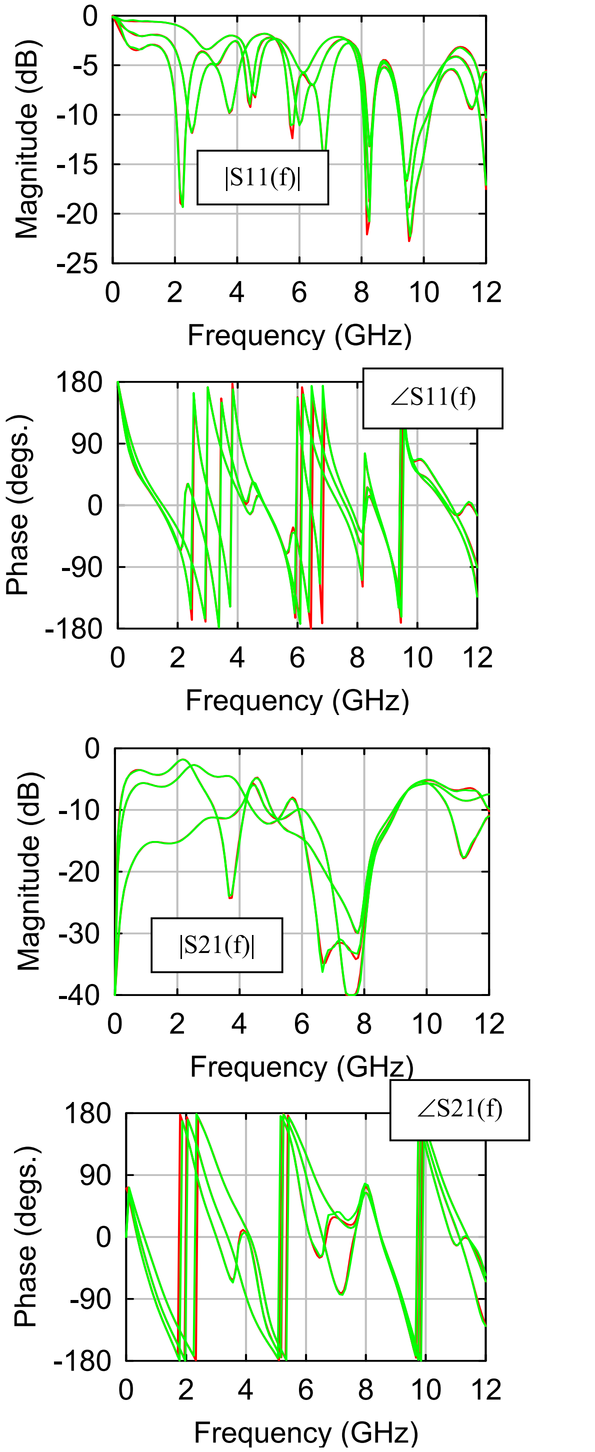

Fig. 4. Frequency domain behaviors predicted by Time-Domain description proposed in this work compared to exact response for three different stub lengths in Fig. 3 (based on 32 FD samples)

The quality of the time-domain representations in Fig. 4 is uniformly excellent: smooth, continuous and exceptionally accurate frequency-domain responses are generated between the much smaller number of frequency-domain points that were used for their initial calculation right up to the maximum frequency f_m. Extensive tests using measured S-data, S-parameters from EM simulation or from circuit simulation has repeatedly confirmed that these results are quite typical of what can be achieved with the method described here. Such small errors that do occur tend to be exactly where one would want them – when the magnitude function is anyway very small or else close to the upper frequency limit of representation f_m.

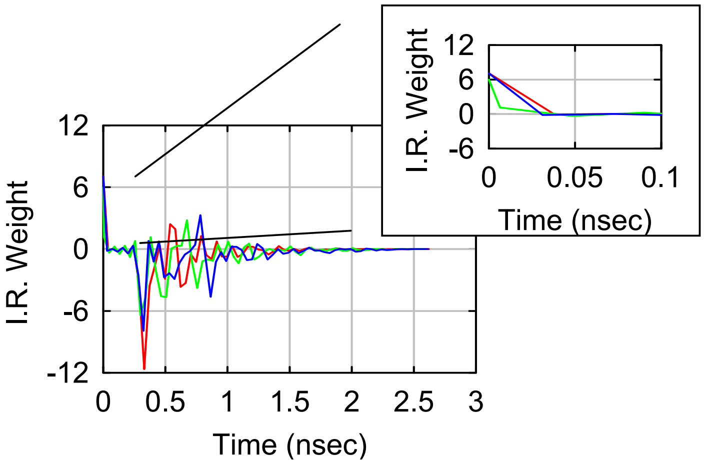

Fig. 5. Time-Domain representation of $S_{11}(f)$ for three line lengths in Fig. 3.

III. Linear and Non-Linear Transient Analysis

We have implemented both linear and non-linear transient simulators using the representation of S-parameter data described in section II. Indeed, the form of discrete-time representation used is very easy to incorporate into such simulators. As an example consider the case where we wish to simulate the transient terminal voltage responses of a linear two-port network known only through its S-parameters when the network is excited at port (1) by a generator $v_{gen}(t)$ that is switched on at time zero. A two-tone form is chosen for $v_{gen}(t)$ as the steady-state response is then easy to determine from standard circuit theory, and this provides a useful verification of the asymptotic trajectories of the transient analyses.

To find the transient responses we must first covert S_{11} and S_{21} into equivalent discrete time representations using the method described in section II. We then need to perform convolutions involving these discrete-time representations with a time-sampled representation of the generator voltage. The convolutions are especially efficient and simple to perform if the time-step for main impulse response record (and ideally also an approximation to the the delay τ) may be chosen to be some integer multiple of the time-step used in the transient analysis Δt. Often this kind of flexibility is available at no particular cost. Note that it is sometimes reported in the literature [1] that convolutions of this kind always have to be carried out over the whole past record from time 0, and therefore the computational cost increases quadratically with time. In fact this need not be true: the impulse responses used here are strictly limited in duration and the cost is low and fixed, independent of the time at which they are computed.

Figure 6 shows an example of a transient analysis performed in this way. We see that the response is smooth and well-behaved and tends in steady-state towards a very close agreement with the analytical results, as required.

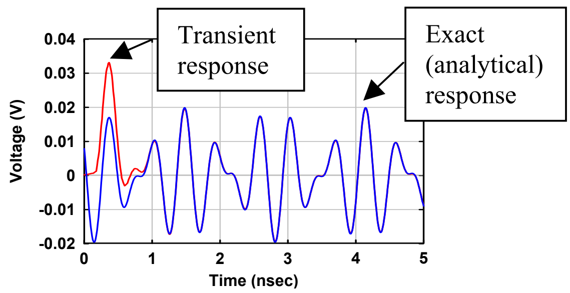

Fig. 6. Comparison between output voltage from convolution-based transient analysis (this method) and steady-state (analytical) result (2.0GHz and 2.7GHz);

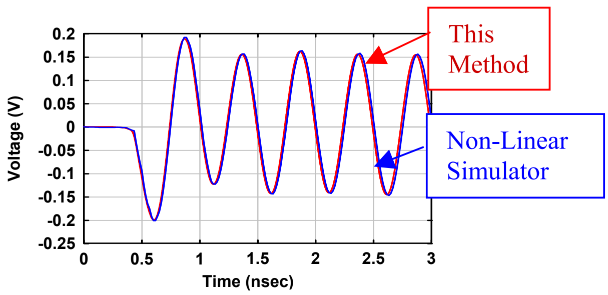

Fig. 7. Comparison between output voltage from this method and output from non-linear simulator at small input amplitude (2GHz)

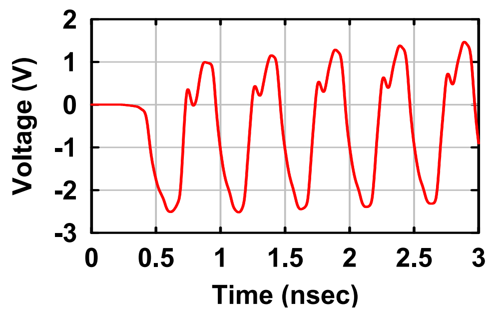

Fig. 8. Output of non-linear simulator based on method described here for amplifier in Fig. 7 at 2GHz and input drive level of 2.5V.

A full non-linear simulation using the approach described in this work for the representation of general matching/bias linear circuit blocks has also been successfully carried out.

As a specific verification of the transient portion of the response, a complete pHEMT amplifier with lossy, distributed matching networks has been described within this simulator. This has been used to derive a table of small-signal S-parameter data for the complete amplifier, which are then converted to the time-domain and a transient analysis performed as just described. These results can be directly compared with a transient analysis of the original circuit using the non-linear simulator (with a small amplitude), providing an independent verification test. These results shown in Fig. 7 again show extremely close agreement. Finally, a highly non-linear analysis was performed for the same amplifier this time with an amplitude of 2.5V. The results shown in Fig. 8, when continued for over one million time steps (with 4 convolutions at each time step), took less than 30 seconds on a 1.3GHz PC.

IV. Discussion and Conclusions

A natural question is what does the form of the representation shown in Fig. 2 do *outside* the frequency range up to f_m for which it was developed. It turns out that it predicts smooth behavior at higher frequencies but of course the behavior is generally quite different from that predicted by the original process. Hence f_m should be chosen to be of the order of the highest significant signal spectral component that is of interest, but this is not particularly critical. We have carefully examined the issue of stability, by generating very long transient responses with exciting signals both below and well above f_m for a wide variety of test circuits. In no case has any evidence of instability been detected.

In conclusion, an effective general method has been developed for the incorporation of frequency-domain (S-parameter) data into a linear or non-linear time-domain transient simulation, in a way that is easy to implement, very accurate, fast and stable.

References

[1] J E. Schutt-Aine, "Transient analysis of non-uniform transmission lines," *IEEE Trans. Circuits and Systems Part I* vol. 39, no. 5, pp. 378--385, May 1992.
[2] Q. Chu, Y. Lau and F. Y. Chang, "Transient analysis of microwave active circuits based on time-domain characteristic models, *IEEE Trans. Microwave Theory Tech.*, vol. 46, pp. 1097-1104, August, 1998
[3] R. Achar and M.S. Nakhla, "Simulation of high speed interconnects", *Proc. IEEE*, vol. 89, pp. 693-728, May 2001
[4] R. Mohan, M. J. Choi et al. "Causal reduced-order modeling of distributed structures in a transient circuit simulator," *IEEE Trans. Microwave Theory Tech.*, vol. 52, no. 9. pp. 2207-2214, September 2004.