RF Power Amplifier Behavioral Modeling Using Volterra Expansion with Laguerre Functions

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***Abstract* — The requirement for an estimation of a large number of parameters is a major limitation in using the Volterra series to model nonlinear RF power amplifiers. In this paper, we propose a new behavioral model for power amplifiers by projecting the classical Volterra series onto a set of Orthonormal Basis Functions (Laguerre functions). This approach enables a substantial reduction in the number of parameters involved, and allows the reproduction of both transient and steady-state behavior of power amplifiers with excellent accuracy.**

***Index Terms* — Volterra series, Laguerre functions, power amplifiers, behavioral modeling.**

I. INTRODUCTION

Behavioral modeling of RF/microwave circuits and systems has received much attention from many researchers in recent years [1]. In behavioral modeling, the nonlinear component is generally considered as a "black-box", which is completely characterized by external responses, i.e., in terms of input and output signals, through the use of relatively simple mathematical expressions. Behavioral modeling techniques provide a convenient and efficient means to predict system-level performance without the computational complexity of full circuit simulation or physical level analysis of nonlinear systems, thereby significantly speeding up the analysis process.

RF power amplifiers (PA) play a crucial role in wireless communication systems. Up to date, most PA behavioral models have been based on AM/AM and AM/PM conversions or polynomial memoryless models, or else suboptimal approximate systems. These methods are not sufficiently accurate for future wideband communication systems, especially with complex modulated signal systems. This is because the output response of the power amplifier at a given instant depends not only on the input signal at the same time instant but also on the input signal at preceding instants over a significant duration, leading to so-called "memory effects".

A truncated Volterra series model [2][3] has been used by a number of researchers to describe the relationship between the input and the output of a nonlinear system with memory. Many Volterra based PA behavioral models also have been proposed [4][5]. However, high computational complexity makes methods of this kind impractical in some real cases, e.g., modeling a PA with strong nonlinearities or with long-term memory effects. This is because the number of coefficients, which are needed to be estimated in the model, exponentially increases with the degree of the nonlinearity and with the "memory length" of the system.

In this paper, we present a new Volterra-based modeling technique for wideband RF power amplifiers, by projecting the classical Volterra series to an Orthonormal Basis Function (OBF), namely, the Laguerre function. In this model, the number of parameters is independent of the memory length, and much smaller than that required in classic Volterra models.

The remainder of the paper is organized as follows. The principle of the Volterra expansion with Laguerre functions is first outlined in section II. Then section III gives the model extraction methodology. Application the new behavioral model to a wideband RF power amplifier is described in section IV.

II. VOLTERRA EXPANSION WITH LAGUERRE FUNCTIONS

The Volterra series is extended from nonlinear power series, and combined with linear convolution. A discrete-time truncated Volterra model is generically described as

$$\begin{aligned} y(n) &= \sum_{m=0}^{M-1} h_1(m)x(n-m) + \sum_{m_1=0}^{M-1}\sum_{m_2=0}^{M-1} h_2(m_1,m_2)x(n-m_1)x(n-m_2) + \cdots \\ &= \sum_{k=1}^{Q}\sum_{m_1=0}^{M-1}\cdots\sum_{m_k=0}^{M-1} h_k(m_1,\cdots,m_k)\prod_{j=1}^{k} x(n-m_j) + e(n) \end{aligned} \quad (1)$$

where $x(n)$ and $y(n)$ is the input and the output of the system, respectively. The $h_k(m_1,\cdots,m_k)$ is the kth order *Volterra kernels*, associated with the system's kth order nonlinearities, and $e(n)$ is the modeling error. Q and M is the truncated order of nonlinearities and the "memory length", respectively. A Volterra model has a clear nonlinearity structure which is a natural extension from a linear impulse response model. Furthermore, the Volterra model is linear in terms of its parameters and hence linear system identification algorithms can be employed to extract the model. However, in this approach, the elements of the Volterra kernels are simply treated as individual parameters, i.e., Dirac impulse responses, to be estimated. This may be not a very efficient description of the expected output since impulse responses tend to decay linearly over time. Therefore the truncated "memory length" M, directly depends on the duration of actual memory in the system. It is thus clear that M must be chosen large enough to include all "memories" which affect the output response of the system. Otherwise the approximation error would become too large and the dynamic representation of the model would be poor. This leads to the huge number of parameters that must be estimated in order to identify the system, which sometimes limits the practical usefulness of the Volterra model.

In linear system identification, Orthonomal Basis Functions (OBF), like the Laguerre functions, have been employed by some researchers as a means to reduce the number of parameters needed for the model construction.

In the Laguerre model, the basis functions, i.e., Dirac impulses, in the FIR filter are replaced by more general and complex orthonormal functions $\{\varphi_k(m)\}$, which decay exponentially to zero at a controllable rate. The discrete time Laguerre functions $\{\varphi_k(m)\}$ are defined by their Z-transform according to

$$L_k(z,\lambda)=\frac{\sqrt{1-|\lambda|^2}}{1-z^{-1}\lambda}\left\{\frac{-\lambda^*+z^{-1}}{1-z^{-1}\lambda}\right\}^k \qquad k\geq 0 \tag{2}$$

where λ is the pole of the Laguerre functions and $|\lambda|<1$, and $(\bullet)^*$ represents the conjugate transpose. Thus, a linear model based on the Laguerre functions can be described as follows

$$y(n)=\sum_{k=0}^{L-1} b_k L_k(z,\lambda)x(n) \tag{3}$$

where b_k is the kth regression coefficients, and $L_k(z,\lambda)$ is the kth discrete Laguerre function given by (2). Note that when $\lambda=0$, the resulting filter is the common transversal FIR filter.

The linear Laguerre model can be re-written as

$$y(n)=\sum_{k=0}^{L-1} b_k l_k(n) \tag{4}$$

where $l_k(n)$ is defined as

$$l_0(n)\triangleq L_0(z,\lambda)x(n) \tag{5}$$

$$l_k(n)\triangleq B(z,\lambda)l_{k-1}(n) \qquad k=1,\ldots,L-1 \tag{6}$$

where

$$L_0(z,\lambda)=\frac{\sqrt{1-|\lambda|^2}}{1-z^{-1}\lambda} \tag{7}$$

$$B(z,\lambda)=\frac{-\lambda^*+z^{-1}}{1-z^{-1}\lambda} \tag{8}$$

An implementation of the Laguerre filter is depicted in Fig. 1.

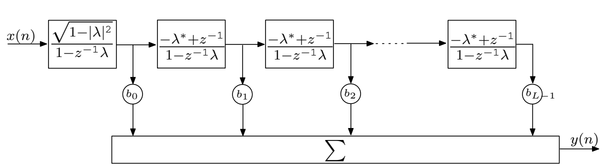

Fig.1. Laguerre filter of order L

From Fig.1, the Laguerre model (3) can be interpreted as discrete filters where the first section represents a first-order low pass filter $L_0(z,\lambda)$, followed by $(k-1)$ all-pass sections $B(z,\lambda)$. Since the Laguerre functions are orthogonalized exponentials, unlike the transversal FIR structure, the parameters $\{b_k\}$ in (4) do not depend on the order L. Laguerre-based models give good performance for systems with low frequency characteristics, e.g., an RF power amplifier with long-term "memory effects".

Based on Laguerre functions, Zheng and Zafiriou [6] expanded the kernels of a Volterra series model in chemical control systems. In this paper, we further extend that idea by formulating the Volterra model using complex Laguerre functions to model an RF power amplifier at the system level.

Consider $x(t)=\Re e\left[\tilde{X}(t)\cdot e^{j\omega_0 t}\right]$ and $y(t)=\Re e\left[\tilde{Y}(t)\cdot e^{j\omega_0 t}\right]$ as the input and output signal of a power amplifier, where ω_0 is carrier frequency and $\tilde{X}(t)$ and $\tilde{Y}(t)$ represents the complex-valued envelopes of the input and output signal, respectively.

Using A/D conversion, a discrete time-domain finite-memory complex baseband Volterra model has the form:

$$\begin{aligned}\tilde{Y}(n)=&\sum_{i=0}^{M-1} h_1(i)\times\tilde{X}(n-i)\\&+\sum_{i_1=0}^{M-1}\sum_{i_2=i_1}^{M-1}\sum_{i_3=0}^{M-1} h_3(i_1,i_2,i_3)\times\tilde{X}(n-i_1)\tilde{X}(n-i_2)\tilde{X}^*(n-i_3)+\cdots\end{aligned} \tag{9}$$

where $h_l(i_1,i_2,\cdots i_l)$ is the lth-order Volterra kernel, M represents the "memory" of the corresponding nonlinearity, $(\bullet)^*$ represents the conjugate transpose. In the above equation, we have removed the redundant items associated with kernel symmetry, and also the even-order kernels, whose effects can be omitted in band-limited modulation systems.

Assuming that the Volterra kernels $h_l(i_1,i_2,\cdots i_l)$ in (9) have a fading memory, i.e., they are absolutely summable on the system memory $[0,M]$, then they can be approximated by a complete basis $\{\varphi_k(m)\}$ of Laguerre functions defined over $[0,L]$ [6]. For instance,

$$h_1(i)=\sum_{k=0}^{L-1} c_1(k)\varphi_k(i) \tag{10}$$

$$h_3(i_1,i_2,i_3)=\sum_{k_1=0}^{L-1}\sum_{k_2=k_1}^{L-1}\sum_{k_3=0}^{L-1} c_3(k_1,k_2,k_3)\varphi_{k_1}(i_1)\varphi_{k_2}(i_2)\varphi_{k_3}{}^*(i_3) \tag{11}$$

are the expansions for the first- and third-order kernels. These expansions are extended to all kernels present in the system. Then the Volterra model becomes

$$\begin{aligned}\tilde{Y}(n)=&\sum_{k=0}^{L-1} c_1(k)l_k(n)\\&+\sum_{k_1=0}^{L-1}\sum_{k_2=k_1}^{L-1}\sum_{k_3=0}^{L-1} c_3(k_1,k_2,k_3)l_{k_1}(n)l_{k_2}(n)l_{k_3}{}^*(n)+\cdots\end{aligned} \tag{12}$$

where

$$l_k(n)=\sum_{m=0}^{M-1}\varphi_k(m)\tilde{X}(n-m)=L_k(z,\lambda)\tilde{X}(n) \tag{13}$$

and $c_p(k_1,k_2,\cdots,k_p)$ are the kernel expansion coefficients.

Note that the variable $l_k(n)$ is a weighted sum of the input epoch values (i.e., discrete convolution). It is obvious that the accuracy of the model depends on the number, L, of the basis functions. However, when compared with using the classic

Volterra model, an appropriate selection of the Laguerre basis functions provides a great reduction in the number of parameters needed to model a nonlinear system to achieve the same accuracy. Implementing the nonlinear Laguerre model is easily achieved by means of a "nonlinear combiner" summing all weighted product term combinations, as shown in Fig.2.

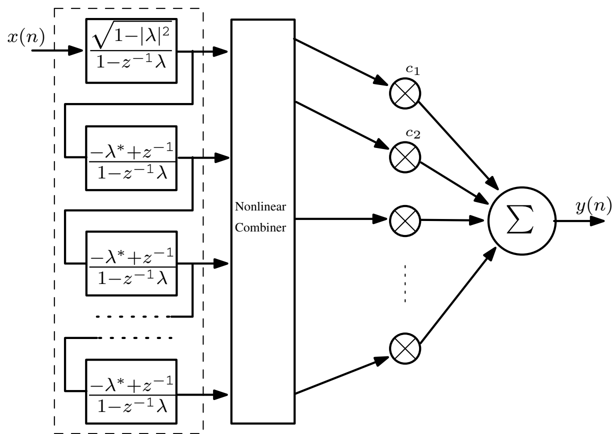

Fig.2. Structure of a Volterra-Laguerre model

III. Model Extraction

As far as the model structure is decided, two important issues are: the selection of the parameters λ and L. In relation to the number of functions, the ideal selection of L is the one that leads the truncation error to be equal to or tending to zero. However, in practice this selection depends on the complexity of the system and it is possible to increase the model quality by increasing the number of functions. The choice of the orthonormal basis pole λ is not critical since the basis is complete for all λ. However, an adequate choice can lead to a more efficient representation of the system since the better the pole choice, the faster will be the convergence of the series and the number of functions can be decreased. Usually, the pole is selected using *a priori* knowledge of the dominant dynamic of the system, such as, for example, considering the shape of its time or frequency response. How to optimize the pole is outside the scope of this work, and we follow the criterion proposed in [7] for Laguerre function pole computation.

When the pole λ is determined, the next step is to find the coefficients $c_p(k_1, k_2, \cdots, k_p)$ in (12). As in a Volterra model, the output of the Laguerre based model is linear respect to the coefficients $c_p(k_1, k_2, \cdots, k_p)$. Consequently, one possible approach to the model parameter estimation problem is to treat it as a large but standard regression problem. In particular, we could form a single large parameter vector $\boldsymbol{\theta}$ containing all of the unknown $c_p(k_1, k_2, \cdots, k_p)$ and define the matrix $\mathbf{X}$ containing all of the product terms $l_{k_1}(n) l_{k_2}(n) \cdots l_{k_p}{}^*(n)$ appearing in the model for $n = 1, \cdots, N$, where N is the total length of the available data record. With these definitions, the Volterra-Laguerre model is re-written as

$$\mathbf{y} = \mathbf{X}\boldsymbol{\theta} + \mathbf{e} \tag{14}$$

where $\mathbf{y} = [\tilde{Y}(1), \cdots, \tilde{Y}(N)]^H$ and $\mathbf{e} = [e(1), \cdots, e(N)]^H$, where $e(k) = d(k) - \tilde{Y}(k)$, $d(k)$ is the desired output and $(\bullet)^H$ represents the Hermitian transpose.

A popular solution to this problem is the *least squares* method, in which $\boldsymbol{\theta}$ is estimated as that value $\hat{\boldsymbol{\theta}}$ minimizing the model error criterion

$$J(\theta) = \sum_{k=1}^{N} |e(k)|^2 = \mathbf{e}^H \mathbf{e} \tag{15}$$

It is a standard result that the estimate minimizing this criterion is:

$$\hat{\boldsymbol{\theta}}(n) = (\mathbf{X}^H \mathbf{X})^{-1} \mathbf{X}^H \mathbf{d} \tag{16}$$

where $\mathbf{d} = [d(1), \cdots, d(N)]^H$.

Other linear adaptive techniques, including the RLS (Recursive Least Squares) and the LMS (Least Mean Squares) algorithms, also can be employed to estimate the model parameters.

IV. Model Validation

In order to validate the proposed behavioral model, we test a class AB medium power amplifier, which has noticeable memory. This power amplifier is operated at 2.14 GHz and excited by downlink 3GPP W-CDMA signals of 3.84 Mcps chip rate and peak-to-average power ratio equal to 6.0 dB @ 0.01% probability on CCDF. The test bench setup uses the ADS-ESG-VSA connected solution from Agilent Technologies [8]. The baseband I/Q signals are generated from Agilent ADS software running on a PC, and downloaded to an Agilent E4438C ESG vector signal generator. This test signal is then passes through the DUT and into an Agilent E4406A vector signal analyzer (VSA). The DUT output test signal is then read from the E4406A VSA back into the ADS simulation environment using the Agilent 89601A VSA software, which is dynamically linked from within ADS. Around 5000 sampling data points are captured from the input and output envelope signals of the PA.

In this example, we truncate the Volterra model to fifth order. After optimization, we choose λ=0.2 and L=3. Only 81 parameters are needed to be estimated.

A comparison between the actual measured data and the waveform computed by using the Volterra-Laguerre behavioral model is presented in Fig. 3. It can be seen that these two sets of data are very close. The average NMSE (*normalized mean square error*) is up to -37.5 dB, the maximum relative error is 0.05 %. To achieve the same accuracy, the classic Volterra models use 244 to 605 parameters [5]. A substantial reduction of the number of model parameters is obtained by an appropriate selection of the orthonormal functions.

The frequency-domain spectra of the power amplifier output signal to W-CDMA excitation is shown in Fig. 4. The ACPR performance are given in Table I. Compared to the measured results, the superior prediction of the amplifier nonlinear performance by the new model is clearly visible.

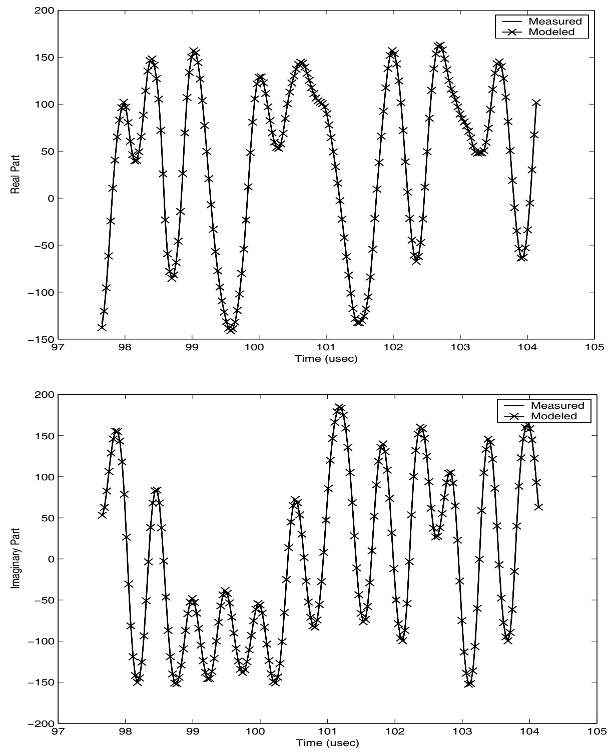

Fig.3. Sample of time domain waveform

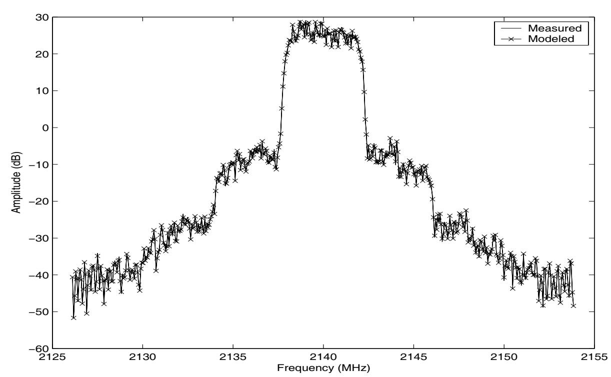

Fig.4. Spectra of the PA output

Table I. Measured and modeled ACPR performance

Performance	ACPR(dBC) (+/- 5MHz)	ACPR (dBC) (+/- 10MHz)
Measured	-36.8/-35.9	-57.1/-56.0
Modeled	-36.7/-36.1	-57.0/-56.4

V. Conclusion

The advantage in using a Volterra-Laguerre expansion model over the conventional Volterra series model lies in the fact that the the number of parameters in a Volterra-Laguerre model is independent of the system memory M , and the order of Laguerre functions L is usually much smaller than M . Therefore, the number of parameters to be estimated can be reduced significantly.

This efficient Volterra-Laguerre based modeling technique can accurately reproduce nonlinear distortions of a power amplifier, including memory effects, allowing use of this modeling approach under wideband complex modulated signal applications. The extraction of the proposed model is simple and affordable either through circuit-level simulation or through calibrated time-domain envelope measurements. The model can also be readily embedded in most commercial CAD environments.

Acknowledgement

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