# Distortion Evaluation of RF Power Amplifiers Using Dynamic Deviation Reduction Based Volterra Series

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*Abstract* — **In this paper, we present a systematic way to evaluate nonlinear distortions induced by RF power amplifiers. This approach is derived from a previously-introduced *dynamic deviation reduction* based Volterra series, in which static nonlinearities and different orders of dynamic behavior can be separately identified. New figures of merit are introduced to evaluate the effects of different order dynamics upon the model accuracy and the performance of PA linearizers, which provides a judicious way to trade off between model complexity and model accuracy.**

*Index Terms* — **Behavioral model, power amplifiers, Volterra series.**

## I. Introduction

Behavioral modeling of RF power amplifiers (PAs) has received much attention from many researchers, and several behavioral models have been developed in recent years. However, the models proposed to date have been mainly based on a pure “black-box” approach, in which all nonlinear distortions induced by the PA are treated in an indistinct way. This sometimes can result in an inefficient model since the distortions can arise from different sources in a PA, and they may affect the output differently. A pure “black-box” approach may use many more parameters than the minimum required for certain model accuracy. In another application field, linearization techniques have been proposed to compensate for the PA’s nonlinear distortions. It is desirable to know which effects are dominant and then to concentrate to remove them; otherwise the linearization will not be efficient or effective.

A modified Volterra series was proposed in [1][2]. This series has the important property that it separates the purely static effects from the dynamic ones, which enables us to classify the PA’s dynamics in different orders and evaluate them separately. However, direct extracting the coefficients of this model involves complicated measurement procedures because this modified series loses the property of linearity in model parameters. Fortunately, in [3][4], the authors extended the Modified Volterra Series to the discrete time domain, and rewrote it in the classical format after dynamic-order truncation. In that new format of representation, the input elements of the Volterra series are organized according to the order of dynamics involved in the system, which is similar to what we had seen in the Modified Volterra Series. However, the property of linearity in the parameters of the model is retained as in the classical Volterra series. This model can therefore be easily extracted directly from measured time-domain of input and output samples of an amplifier by employing simple linear system identification algorithms.

In this paper, we further extend this Volterra model and use it to derive a new distortion evaluation algorithm. It can be employed to quantify the effects of different types of distortions in a PA, so that we can effectively trade off between model complexity and model fidelity in a judicious manner in practical applications.

## II. Model Description

A Volterra series is a combination of linear convolution and a nonlinear power series. It provides a general way to model a time-invariant, causal and stable nonlinear dynamic system with fading memory, so that it can be employed to describe the relationship between the input and the output of an amplifier with memory. However, high computational complexity continues to make methods of this kind rather impractical in some real applications because the number of parameters to be estimated increases exponentially with the degree of nonlinearity and memory length of the system.

To overcome the complexity of the general Volterra series, a new format of representation for the Volterra series was proposed in [3] as follows,

$$\begin{aligned} y(n) = &\sum_{p=1}^{P} h_{p,0}(0,\ldots,0)x^{p}(n) \\ &+\sum_{p=1}^{P}\{\sum_{r=1}^{p}[x^{p-r}(n)\sum_{i_1=1}^{M}\cdots\sum_{i_r=i_{r-1}}^{M} \\ &\cdot h_{p,r}(0,\cdots,0,i_1,\cdots,i_r)\prod_{j=1}^{r}x(n-i_j)]\} \end{aligned} \quad (1)$$

where $x(n)$ and $y(n)$ represents the input and the output, respectively, and $h_{p,r}(0,\ldots,0,i_1,\ldots,i_r)$ represents the $p$th order Volterra kernel in which the first $p$-$r$ indexes are “0”, corresponding to the input item $x^{p-r}(n)x(n-i_1)\cdots x(n-i_r)$. In real applications, the Volterra series is normally truncated to finite nonlinear order $P$ and finite memory length $M$.

Different from the classical Volterra series, in (1), the input elements are organized according to the order of dynamics involved in the model, namely, $r$ represents the order of the dynamics of the input products. Based on this new representation, an effective model order reduction method was proposed, called *dynamic deviation reduction*, in which higher order dynamics can be removed, by controlling the value of $r$,

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since it is assumed that the effects of nonlinear dynamics tend to fade with increasing order in many real power amplifiers. As shown in [3], this leads to significant simplification of the model structure. Furthermore, it was also shown that the new representation of Volterra series in (1) is equivalent to the Modified Volterra Series, which means that the coefficients of the Modified Volterra Series can be directly calculated from the coefficients extracted for (1).

In the Modified Volterra series [1], by introducing a dynamic deviation function $e(n,i)$,

$$e(n,i) = x(n-i) - x(n) \tag{2}$$

the output signal $y(n)$ in (1) can be expressed as:

$$y(n) = y_s(n) + y_d(n) \tag{3}$$

where $y_s(n)$ and $y_d(n)$ represents the static and dynamic characteristics of the system, respectively. $y_s(n)$ can be expressed as a power series of the current input signal $x(n)$:

$$y_s(n) = \sum_{p=1}^{P} a_p x^p(n) \tag{4}$$

where $a_p$ is the coefficient of the polynomial function, while $y_d(n)$ is the purely-dynamic part:

$$y_d(n) = \sum_{r=1}^{R} y_{d,r}(n) \tag{5}$$

where $y_{d,r}(n)$ represents $r$th-order dynamics, which can be formulated as

$$y_{d,r}(n) = \sum_{p=1}^{P} x^{p-r}(n) \cdot \sum_{i_1=0}^{M} \cdots \sum_{i_r=0}^{M} w_{p,r}(i_1,\cdots,i_r) \prod_{j=1}^{r} e(n,i_j) \tag{6}$$

where $w_{p,r}(\cdot)$ represents the $r$th-order dynamic kernel of the $p$th order nonlinearity [3]. The coefficients of the dynamic series, $a_p$ and $w_{p,r}(\cdot)$, can be directly derived from the coefficients of the Volterra formulation in (1), $h_{p,r}(0,\ldots,0,i_1,\ldots,i_r)$ [3].

In the Modified Volterra series above, all linear components are still mixed in the output, i.e., both $y_s(n)$ and $y_d(n)$ include linear and nonlinear components. However, in a real application, e.g., linearization of a PA, the linear gain should be separated from the nonlinear distortion since only the nonlinear distortions need to be compensated. In this paper, we identify the linear transfer function of the PA using the input-output cross-correlation techniques in the discrete frequency domain.

According to the theory of linear system identification [5], the best linear approximation of a system can be estimated when it is excited by a pre-determined excitation. In that way, the linear transfer function $\overline{H}_1(\omega)$ is given by the ratio of the cross-spectral power density function between the output and the input, $S_{yx}(\omega)$, to the auto-spectral power density function of the input, $S_{xx}(\omega)$:

$$\overline{H}_1(\omega) = \frac{S_{yx}(\omega)}{S_{xx}(\omega)} = \frac{\langle Y(\omega).X(\omega)^* \rangle}{\langle X(\omega).X(\omega)^* \rangle} \tag{7}$$

The auto- and cross-spectral densities can be estimated from the synchronous acquisition of a number, $N$, of records of fixed length of the input and the output, i.e.,

$$S_{yx}(\omega) = \frac{1}{N} \sum_{k=1}^{N} Y_k(\omega).X_k(\omega)^* \tag{8}$$

and

$$S_{xx}(\omega) = \frac{1}{N} \sum_{k=1}^{N} X_k(\omega) X_k(\omega)^* \tag{9}$$

where $X_k(\omega)$ and $Y_k(\omega)$ is the DFT of the input and the output, respectively. The linear gain of the system, $G$, is then equal to $\overline{H}_1(0)$, and the static linear part of the output becomes

$$y_{s,L}(n) = G \cdot x(n) \tag{10}$$

and thus the nonlinear static part is

$$y_{s,NL}(n) = y_s(n) - G \cdot x(n) \tag{11}$$

Finally, the output of the PA, $y(n)$, can be represented by

$$y(n) = y_{s,L}(n) + y_{s,NL}(n) + \sum_{r=1}^{R} y_{d,r}(n) \tag{12}$$

in which the different contributions of the output are separated.

Note that, for formulation simplicity, only a real RF signal is considered in the derivations above. Normally, in wireless system analysis and design, most simulators often use baseband complex envelope signals to evaluate the system performance since modulation techniques are normally employed in modern wireless communication systems, where only the envelopes carry the useful information. To handle these carrier-modulated signals, the Volterra model above has to be transformed to the low-pass equivalent format [4].

## III. Distortion Evaluation

As mentioned in the introduction, it is very important to know what are the effects of the different distortions induced by a PA, as this has a determinant impact on the quality of PA behavioral model and the design of a PA linearizer. In the previous section, we have introduced how to separate different distortions from the PA output. In the following, we will discuss how to evaluate their effects and quantify them in a real system.

### *A. Behavioral Modeling*

In power amplifier behavioral modeling, we always need to trade off between model complexity and model accuracy. The higher accuracy often results in higher computational complexity. Although many model-pruning approaches have been proposed to do this trade-off, there is still no systematic way to verify if the model truncation is truly appropriate to the PA under study. Indeed, because behavioral models developed to date have been mainly based on a pure "black-box" approach, all PA distortion components are often treated indistinctly. However, by employing the dynamic derivation reduction based Volterra series, the distortion from static and different order of dynamics can be separately identified, which provides an effective way to evaluate distortions induced by the PA.

To assess the predictive accuracy of the model, *NMSE* (normalized mean square error) is often employed in the discrete time domain, which is the total error vector magnitude power between the measured and modeled waveforms, normalized to the measured signal power. As mentioned earlier, by control the value of $r$ in (1), we can select different order dynamics to be included in the model. In order to improve the model accuracy, the larger value of $r$ needs to be selected, which leads to a more complex model since a larger number of coefficients will be involved. However, an obsolete value of *NMSE* can not tell if it is worth to increase the model complexity by increase the order of dynamics.

To quantify the effects of the different order of dynamics, we define a gain factor of *NMSE* as follows,

$$\eta_{NMSE} = \frac{NMSE - NMSE_{static}}{NMSE_{static}} \times 100\% \quad (13)$$

where $NMSE_{static}$ represents the *NMSE* of the static model of a PA. It can be seen that $\eta_{NMSE}$ shows how much improvement can be achieved after including higher order dynamics in the model. This gives us a clear idea how the different order of dynamics affect the model performance.

### *B. Linearization*

In the frequency domain, *ACPR* (Adjacent Channel Power Ratio) is defined as the power contained in a defined bandwidth at a defined offset from the channel center frequency, with respect to the power within the desired transmission bandwidth. It is one of the most important figures of merit employed to measure the degree of signal spreading into adjacent channels, caused by nonlinearities in a PA excited by modulated signals. In PA linearization, the improvement of *ACPR* is normally used to evaluate the performance of the linearizer.

Similar to $\eta_{NMSE}$, we define a gain factor of *ACPR* as,

$$\eta_{ACPR} = \frac{ACPR - ACPR_0}{ACPR_0} \times 100\% \quad (14)$$

where $ACPR_0$ represents the *ACPR* of the PA without compensation. By comparing the relative *ACPR* improvement, we can justify if it is worth to do the compensation.

For simplicity, we consider the linearizer as a parallel block to the PA. As shown in Fig. 1, a compensation signal is simply added at the output to cancel the distortions. Naturally, such a block can be transferred to the upstream of the PA to operate as a pre-distorter.

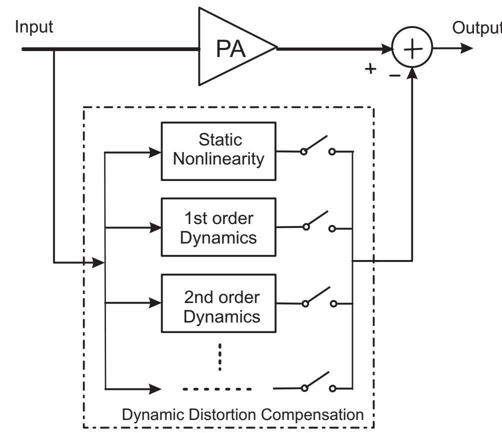


Fig. 1. Dynamic distortion compensation

To investigate the effects of different distortions, we divide the compensation signal into several sub-blocks according to the order of dynamics. Since, in most cases, the static nonlinearities dominate the sources of distortion, we start by compensating static nonlinear distortions and evaluate their effects. Then we move on to address the dynamic parts. By switching on different branches of the blocks, we can observe how the distortion changes in the output.

## IV. RESULTS

To demonstrate how to evaluate and compensate different distortion characteristics in a real system, we tested two power amplifiers. The first one, PA1, mainly presented static nonlinearity while the second one, PA2, had significant memory effects. Both PAs were operated at 950 MHz and excited by CDMA2000 signals with 1.2288 Mcps chip rate. The experimental test bench setup used the ADS-ESG-VSA connected solution [3]. Around 12,000 sampling data points, with a sampling rate of 4.9152 MHz, were captured from the PA input and output envelope signals.

The nonlinearity order $P$ was set to five and memory length $M$=3. By selecting the value of $r$, from 0 to 2, three models with different dynamic-order truncation were extracted for both PAs, respectively. The NMSE performances of these models are shown in Table I, where we can see that in the PA1 the static model performed quite well and increasing the dynamic orders did not improve much of the accuracy. For instance, only 16.2% of $\eta_{NMSE}$ was gained while 51 more parameters were added when the second order dynamics were included. However, in the PA2 the model fidelity was significantly improved when higher order dynamics were taken into account, e.g.., $\eta_{NMSE}$ =69.9% for the 2$^{nd}$ order model.

TABLE I

MODEL PERFORMANCE IN THE TIME DOMAIN

| | PA1 | PA2 | PA1 | PA2 | PA1 | PA2 |
|---|---|---|---|---|---|---|
| Order of dynamics | static | | static, 1st | | static, 1st, 2nd | |
| No. of coefficients | 3 | | 18 | | 54 | |
| *NMSE* (dB) | -26.5 | -18.3 | -28.4 | -25.1 | -30.8 | -31.1 |
| $\eta_{NMSE}$ (%) | - | - | 7.2 | 42.6 | 16.2 | 69.9 |

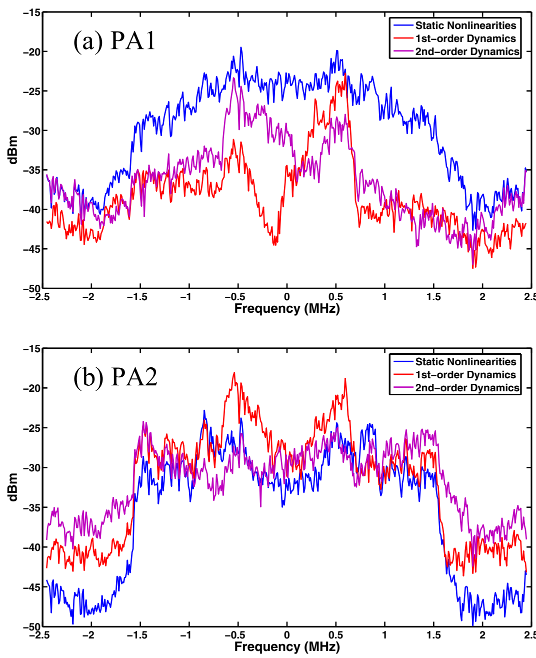


Fig. 2. Spectra of the distortions

TABLE II

ACPR PERFORMANCE IN THE FREQUENCY DOMAIN

| | PA1 | PA2 | PA1 | PA2 | PA1 | PA2 |
|---|---|---|---|---|---|---|
| Order of dynamics | static | | static, 1st | | static, 1st, 2nd | |
| *ACPR* (lower/upper) dBc | -40.2 / -41.0 | -34.1 / -36.2 | -42.1 / -42.0 | -39.0 / -39.5 | -43.4 / -44.0 | -42.9 / -43.8 |
| $\eta_{ACPR}$ (%) | 40.0 / 43.3 | 20.9 / 18.7 | 46.7 / 46.8 | 38.2 / 29.5 | 51.2 / 53.8 | 52.1 / 43.6 |

*ACPR without compensation:
PA1:-28.7/-28.6 dBc and PA2: -28.2/-30.5 dBc

To evaluate the effects of different dynamics upon the PA linearization, we first calculated the coefficients of the equivalent Modified Volterra series from the coefficients of the dynamic deviation based Volterra models extracted earlier. Then we extracted the linear transfer functions of the PAs using cross-correlation techniques, so that the linear parts can be removed. Finally, the different distortions were separated. The spectra of different distortions are shown in Fig. 2, where we can see that in the PA1 the static nonlinearities dominated the distortion while the dynamic parts presented significant distortion in the PA2.

Table II gives the ACPR performances of the PAs after different distortions were compensated. As expected, in the PA1 the ACPR was significantly improved after removing the static nonlinear distortion but compensating the dynamics did not show obvious benefits. For example, only 2dB improvement of ACPR was achieved after canceling the 1st order dynamics. However, in the PA2 only compensating the static nonlinearities was not good enough since there were significant memory effects induced by the dynamic behavior of the PA.

## V. CONCLUSION

A new distortion evaluation approach for RF power amplifiers has been proposed in this paper. With this method, the effects from different distortions induced by the PA can be quantified and compensated separately. This provides an effective way to trade off between model complexity and model accuracy, which can play a key role in selecting behavioral model structures and designing efficient linearizers for RF power amplifiers.


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