Behavioral Modeling of RF Power Amplifiers Based on Pruned Volterra Series

Anding Zhu, *Member, IEEE,* and Thomas J. Brazil, *Fellow, IEEE*

***Abstract*—Behavioral modeling techniques provide a convenient and efficient means to predict system-level performance without the computational complexity of full circuit simulation or physics-level analysis of nonlinear systems, thereby significantly speeding up the analysis process. General Volterra series based models have been successfully applied for radio frequency (RF) power amplifier (PA) behavioral modeling, but their high complexity tends to limit their applications to "weakly" nonlinear systems. To model a PA with strong nonlinearities and long memory effects, for example, the general Volterra model involves a great number of coefficients. In this letter, we propose a new simplified Volterra series based model for RF power amplifiers by employing a "near-diagonality" pruning algorithm to remove the coefficients which are very small, or else not sensitive to the output error, therefore dramatically reducing the complexity of the behavioral model.**

***Index Terms*—Behavioral model, FIR digital filters, power amplifier, Volterra series.**

I. Introduction

BEHAVIORAL modeling is generally proposed to characterize a complete nonlinear system, or a very large section of such a system, in terms of input and output signals using relatively simple mathematical expressions. In this kind of system-level model, the modeled device is considered as a "black-box," i.e., in principle, no knowledge of the internal structure is required and the modeling information is completely contained in the external responses of the device. Owing to this feature, the parameters of the model can be effectively estimated from measured transient responses or simulated results from detailed reference transistor-level models. Behavioral models are reduced-order models of circuit-level devices, which can provide fast prediction of system performance in top-down designs.

A truncated Volterra series has been successfully used to derive some behavioral models for RF power amplifiers by many researchers in recent years. However, high computational complexity makes standard methods of this kind limited to "weak" nonlinearity [1], [2], e.g., only up to third or fifth order, which is not enough in many practical situations. In this letter, we present a "near-diagonality" structural restriction approach to prune some redundant kernels in the general Volterra series based model. With this method, the number of coefficients of the Volterra model can be dramatically reduced and the structure of the model can be significantly simplified, without substantially compromising the accuracy of the model. This technique allows us to model a PA with higher-order nonlinearities and longer-term memory effects.

Manuscript received April 6, 2004; revised July 12, 2004. This work was supported by Science Foundation Ireland. The review of this letter was arranged by Associate Editor A. Stelzer.

The authors are with the Department of Electronic and Electrical Engineering, University College Dublin, Dublin 4, Ireland (e-mail: anding.zhu@ucd.ie; tom.brazil@ucd.ie).

Digital Object Identifier 10.1109/LMWC.2004.837380

The remainder of the letter is organized as follows. In Section II, we review the background of Volterra series based modeling techniques. In Section III, we introduce a new pruning algorithm to effectively remove redundant coefficients of a PA model, then illustrate measured results in Section IV. A conclusion is contained in Section V.

II. Volterra Series Based Modeling

A truncated Volterra series can be used to represent a wide class of time-invariant nonlinear systems with memory effects. Consider $x(t) = \mathrm{Re}[\tilde{x}(t) \cdot e^{j\omega_0 t}]$ and $y(t) = \mathrm{Re}[\tilde{y}(t) \cdot e^{j\omega_0 t}]$ as the input and output signals of a power amplifier, where ω_0 is carrier frequency and $\tilde{x}(t)$ and $\tilde{y}(t)$ represents the complex-valued envelope of the input and output signal, respectively.

Using A/D conversion, a discrete time-domain finite-memory complex baseband Volterra model of a power amplifier has the form

$$\begin{aligned}\tilde{y}(n) = &\sum_{i=0}^{m_1-1} h_1(i) \times \tilde{x}(n-i)\\ &+ \sum_{i_1=0}^{m_3-1}\sum_{i_2=i_1}^{m_3-1}\sum_{i_3=0}^{m_3-1} h_3(i_1,i_2,i_3)\\ &\times \tilde{x}(n-i_1)\tilde{x}(n-i_2)\tilde{x}^*(n-i_3) + \cdots \qquad (1)\end{aligned}$$

where $h_l(i_1, i_2, \ldots, i_l)$ is the lth-order Volterra kernel, m_l represents the "memory" of the corresponding nonlinearity, and $(\cdot)^*$ represents the conjugate transpose. In the above equation, we have removed the redundant items associated with kernel symmetry, and also the even-order kernels, whose effects can be omitted in band-limited modulation systems. Furthermore, in practical situations, we generally truncate the model to finite order N.

A new Volterra series based behavioral model for power amplifiers has been proposed by authors [3]. In this model, a rearrangement is performed on the elements of the input vector to create a nonrectangular vector, the so-called V-vector [4] (as shown in Fig. 1). The time-shift property can be preserved in this approach, thereby avoiding permutations in the process of model extraction, which significantly reduces the computational complexity. We can define a set of primary signals that carry all the information needed for the estimation of the convolution, corresponding to the first column of the input data V-vector $\tilde{X}(n)$, and then use a linear FIR filter to implement

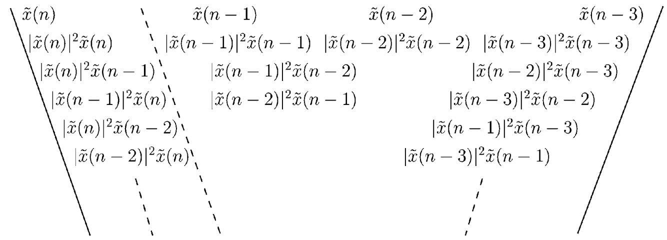

Fig. 1. V-vector of input data.

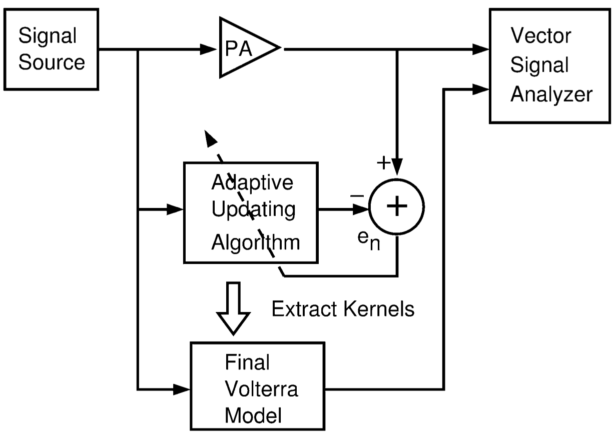

Fig. 2. Block diagram for model extraction.

the convolution for each row of $\tilde{X}(n)$ separately. Summing together all the filter outputs, the final output of the Volterra behavioral model is obtained. The primary signals are actually computed recursively from lower order products. A diagram representing the process of model extraction is shown in Fig. 2.

III. Pruning Algorithm

The fast parallel behavioral modeling technique proposed in [3] significantly improves the data processing speed and saves on computation time. However, this model still inherits the high complexity of general Volterra models because no effort has been made to simplify the underlying model structure. To model a power amplifier with strong nonlinearities and long memory effects, a model of this kind still involves a very large number of filters in the filter-bank, which makes it impractical in some real applications. Fortunately, in practical situations, memory effects in real amplifiers decline with time. This means that in the discrete domain, elements with longer time delay taps in the input vector of the model, have less effect on the output signal, and the coefficients corresponding to them are accordingly very small. It is therefore reasonable to force them to zero during model extraction, a process which is described as pruning [5]. The result is a simplification of the structure of the model and an improvement in simulation speed.

Generally, a brute-force pruning method involves setting individual coefficients to zero and evaluating the change in the error. If the error is increased unacceptably then the coefficient is restored, otherwise it is removed. Probably the simplest nontrivial pruned Volterra model is the diagonal Volterra model, also called the memory polynomial model [6], where all off-diagonal terms are zero. Although this restriction reduces the

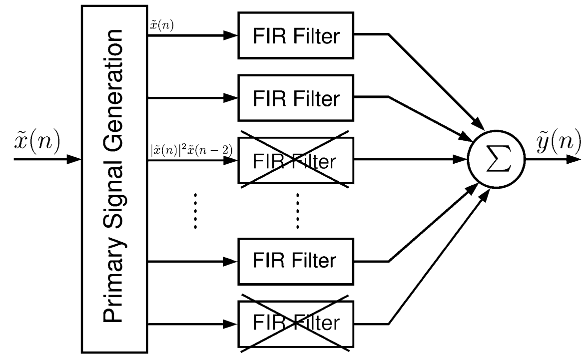

Fig. 3. Pruned Volterra series based behavioral model.

number of model parameters tremendously, it also has significant behavioral consequences, e.g., decreasing fidelity of the model, because, in some cases, the off-diagonal terms are more important than the diagonal terms. As a result, a potentially interesting approach involves relaxing the restriction condition. It may be noted that in the diagonal Volterra model $|i_m - i_n| = 0$ for all $1 \le m, n \le j$, where i_m or i_n represents the memory length of the input signal and j is the maximum memory length. Relaxing this condition to $|i_m - i_n| \le l$ for some small integer l would correspond to imposing a "near-diagonality" structural restriction, giving some increase in flexibility at the expense of a corresponding increase in the number of parameters. This "near-diagonality" structure was used for internal model control (IMC) in chemical engineering applications [7]. Here, we extend the pruning approach to power amplifier behavioral modeling since this kind of solution is quite suitable for the kind of filter-bank-based behavioral model that we have proposed in [3], where coefficients in the same diagonal line of the weight vector fall within the same FIR filter. With the "near-diagonality" structural restriction, i.e., restricting l, the coefficients, which are "far away" from the main diagonal in the model, are removed, then the corresponding FIR filters in the filter-bank can be deleted thereby, as shown in Fig. 3. This approach to restriction has the capability to dramatically simplify the structure of the model and reduce the computational complexity of model extraction.

Furthermore, the "near-diagonality" reduction approach allows us to model a power amplifier with stronger nonlinearity or longer memory effects. As mentioned previously, the general Volterra model is limited to model weakly- or moderately-nonlinear systems, e.g., only reaching to the fifth order for the cases we discussed earlier. When the model order is over seven, the system does not always converge due to large number of coefficients required. However, if we remove many coefficients in each lower-order Volterra model using the "near-diagonality" restriction, the total number of coefficients is reduced, which allows us to add higher order coefficients/kernels into the model to take account of the higher order distortion. Meanwhile, for modeling longer-term memory effects, the number of coefficients increases linearly with respect to the memory length in this new model. This is because no new primary signal, which is in the first column of the input V-vector in Section II, is needed for the new model construction when the l is fixed. To further reduce the complexity, we can choose a different value for l in $|i_m - i_n| \le l$ for different orders, e.g., smaller l for higher order

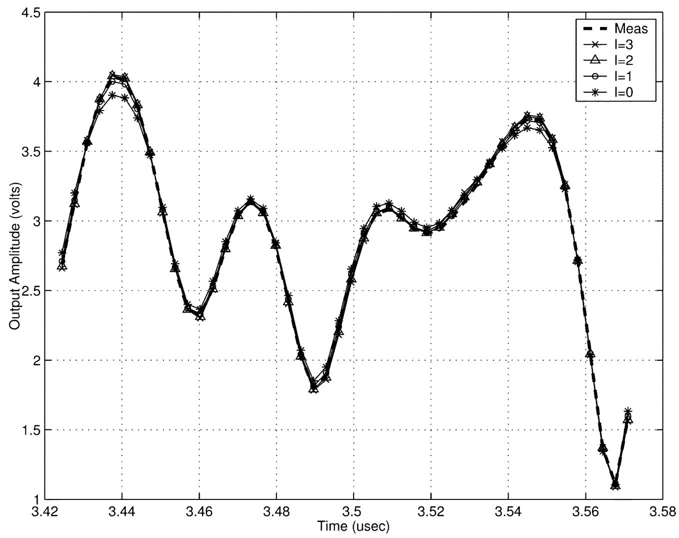

Fig. 4. Sample of time domain waveform.

nonlinearities. How to select l depends on the practical characteristics of power amplifiers and the model fidelity required.

IV. Model Validation

In order to validate the modeling technique proposed, a class AB medium power LDMOS amplifier with noticeable memory effects was used. This PA was operated at 2.14 GHz, and excited by a downlink 3GPP W-CDMA signal with 3.84 Mcps chip rate. The test bench setup used to characterize the amplifier was based on the ADS-ESG-VSA connected solution from Agilent Technologies [8]. In the measurement, we considered the power amplifier as a real "black-box." The baseband I/Q signals were generated from Agilent ADS software running on a PC, and downloaded to an E4438C ESG vector signal generator. These test signals were then passed through the device under test (DUT) and into an Agilent E4406A vector signal analyzer (VSA). The DUT output test signals were read from the E4406A VSA back into the ADS simulation environment using the Agilent 89 601A VSA software, which was dynamically linked from within ADS. Around 2000 sampling data points were gathered from the measured input and output complex envelope signals, which were sent to the fast adaptive filter depicted in Fig. 2. Here, we truncated the Volterra series to fifth order, and memory length to three. Several Volterra-based behavioral models with different restriction conditions were extracted for this power amplifier using in-house software implemented in MATLAB. A sample of the time-domain output envelope waveforms of W-CDMA signals is shown in Fig. 4. Table I gives the performance comparison of the models with different l for the PA we tested. The unrestricted "full" model is the case of $l = 3$ while $l = 0$ leads to the memory polynomial model. The results indicate the model still keeps good performance after some restrictions while the number of coefficients required to be extracted has been significantly reduced. When $l = 1$, for instance, the average relative error between modeled and measured responses is only 0.028%, corresponding normalized mean square error (NMSE) is up to -35.5 dB, but the number of coefficients has dropped from 244 to 54 in this case.

TABLE I
Comparison of the Models With Different l

l	3	2	1	0
number of coefficients	244	133	54	12
Relative error(%)	0.024	0.026	0.028	0.063
NMSE (dB)	-36.1	-35.9	-35.5	-32.0

As mentioned earlier, we can model a power amplifier with stronger nonlinearities and longer term memory effects by employing the "near-diagonality" reduction. For example, in this case, we can select $l = 1$ for third and fifth order, and $l = 0$ for seventh order kernels. We obtain the nearly same NMSE performance as that from the fifth order general model in the time domain, but we can predict the seventh order distortion more accurately in the frequency domain. Also, we may increase the memory length in the model to take account of longer term memory effects induced by the PA.

V. Conclusion

A "near-diagonality" model reduction method for RF power amplifier behavioral modeling has been introduced in this letter. This kind of coefficient-restriction technique leads to simpler model structure, which may significantly reduce the complexity of the Volterra based behavioral model. The agreement obtained between the measured and modeled response proves this approach makes the application of the Volterra model more flexible and effective, in practical situations.

References

[1] M. C. Jeruchim, P. Balaban, and K. S. Shanmugan, *Simulation of Communication Systems*, 2nd ed. Norwell, MA: Kluwer, 2000.

[2] V. J. Mathews and G. L. Sicuranza, *Polynomial Signal Processing*. New York: Wiley, 2000.

[3] A. Zhu, M. Wren, and T. J. Brazil, "An efficient Volterra-based behavioral model for wideband RF power amplifiers," in *IEEE MTT-S Dig.*, vol. 2, 2003, pp. 787–790.

[4] A. Carini, E. Mumolo, and G. L. Sicuranza, "V-vector algebra and its application to Volterra-adaptive filtering," *IEEE Trans. Circuits Syst. II*, vol. 46, pp. 585–598, May 1999.

[5] R. Reed, "Pruning algorithms-a survey," *IEEE Trans. Neural Networks*, vol. 4, pp. 740–747, Sept. 1993.

[6] J. Kim and K. Konstantinou, "Digital predistortion of wideband signals based on power amplifier model with memory," *Electron. Lett.*, vol. 37, no. 23, pp. 1417–1418, 2001.

[7] F. J. Doyle III, R. K. Pearson, and B. A. Ogunnaike, *Identification and Control Using Volterra Models*. New York: Springer-Verlag, 2001.

[8] "Connected simulation and test solutions using the advanced design system," Tech. Rep., Agilent Technologies, Applicat. Notes, No. 1394, 2000.