Peak to Average Power Ratio Reduction Technique for OFDM Using Pilot Tones and Unused Carriers

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***Abstract* – One of major drawbacks of Orthogonal Frequency Division Multiplexing (OFDM) is the high Peak-to-Average Power Ratio (PAPR) of the transmit signal. Due to the non-linear characteristics of the power amplifier (PA), significant out-of-band and in-band distortions occur when high peak signals exceed the saturation level of the PA. In this paper we present a new PAPR reduction technique which exploits the use of unused-carriers as well as the phase information of pilot signals in OFDM systems to reduce the PAPR while not degrading channel estimation or frequency offset. Compared to conventional techniques such as clipping and windowing, this technique introduces much less out-of-band distortions and provides a lower bit-error-rate (BER) since there is no interference to the original data signals. There is also no requirement for side information to be transmitted to the receiver.**

***Index Terms* – PAPR, HPA, OFDM, non-linearity, clipping**

I. Introduction

OFDM is a popular modulation technique which has many well documented advantages such as high spectral efficiency, immunity to frequency selective fading and high data rate transmission [1-5]. Today OFDM has been widely employed in Asymmetric Digital Subscriber Line (ADSL), Digital Audio Broadcasting (DAB), Digital Video Broadcasting (DVB), Local Area Network (LAN) and other high-speed data application for both wireless and wired communications.

However, one of major drawbacks of OFDM is the high peak-to-average power ratio of the transmit signal which occurs due to the summation of many subcarrier-modulated signals. A high PAPR requires a wide dynamic range for the power amplifier at the transmitter, or more commonly the power amplifier needs to be backed off to accommodate high peaks. This results in significant reduction of the transmission power which leads to very low power efficiency. For example, in the IEEE 802.11a system, the typical power efficiency of a class AB PA is only 18% [6]. It is therefore preferable if possible to reduce the PAPR of the signal to avoid the use of back-off.

Several techniques have been developed to address the PAPR problem [7]. Clipping is the simplest technique for reducing the PAPR, however it causes both in-band and out of-band distortion. Filtering can be employed to alleviate out-of-band distortion but results in significant peak re-growth. Repeated clipping and filtering can lead to serious degradation in BER [8]. Windowing is another approach that offers reduced out-of-band radiation, but the window has to be as narrow as possible in the frequency domain and the impulse response in the time domain should not last too long, otherwise more signal samples are affected, which increases the BER [9].

Tone reservation developed by Tellado [10] is also an effective technique for reducing the PAPR of OFDM signals but causes a reduction in data through-put as data carriers are used to generate an effective cancellation signal in the time domain to reduce high peaks.

In this paper, we present a novel PAPR reduction technique that results in very low out-of-band distortion, requires no side information to be transmitted to the receiver and has a lower BER compared with clipping, recursive clipping and filtering and windowing. This is achieved through using the unused sub-carriers and the phase information of pilot tones in the OFDM systems to optimize the signal energy distribution and therefore reduce the high peaks. This technique has been verified with IEEE802.11a, IEEE 802.16e (WiMax), and DVB systems.

II. System Overview

For an OFDM system with N sub-carriers, the baseband signal in discrete form can be written as

$$x(t)=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}X_k e^{j(2\pi k\Delta f t)},0\le t\le T \tag{1}$$

where X_k is the symbol carried by the kth sub-carrier, Δf is the frequency difference between sub-carriers, and T is the OFDM symbol duration. Since the signal which passes through the power amplifier, is in the continuous time domain, which can be significantly higher than the discrete-time estimate, it is necessary to oversample the signal by a factor of at least 4 to enable accurate peak detection [11]. For an oversampling factor of I, the input signal to the IFFT is extended by including $N(I-1)$ zeros in the centre of the signal.

The PAPR of the transmitted signal can be expressed as:

$$PAPR(x)=10\log_{10}\left(\frac{\max|x(n)|^2}{E\left[|x(n)|^2\right]}\right) \tag{2}$$

where $|x(n)|$ returns the magnitude of $x(n)$, and $E[\cdot]$ denotes the expectation operation. As more sub-carriers are added, higher peak values may occur, hence the PAPR increases proportionally with the number of sub-carriers.

In practical OFDM-based applications, not all subcarriers are used to transmit the information data. Some subcarriers are set to zero to prevent out-of-band radiation. For example, in an IEEE802.11a system, 64 tones are employed, in which 48 tones are data carriers, 4 carriers are pilot tones and the remaining 12 tones have a value of zero and are unused. Since these unused tones do not affect the original data carriers, they can be used to reduce the PAPR without increasing the BER or a reduction in data through-put. Although this results in a slight broadening of the original spectrum, it is possible for the spectrum to remain well inside the spectrum mask defined in the standards. Wang et al. [12-13] proposed a PAPR reduction technique through use of the unused tones in IEEE 802.11a wireless local area network (WLAN) and the terrestrial digital video broadcasting (DVB) systems. However, an iterative exhaustive search for suitable values for the unused tones has to be employed. Also the algorithm must ensure that the peak value does not increase at each iteration. Gatherer et al. [14] also propose a PAPR reduction technique using unused tones which will be compared with the proposed new technique in section III.

In OFDM systems, pilot tones are normally employed to enable carrier frequency offset and channel estimation. In order to minimize the channel mean square error, the pilot tones must be equidistant and equi-powered [15]. For optimal channel estimation, all pilot tones must have the same amplitude, but there are no particular requirements for the phase. Hosokawa et al. [16] proposed a PAPR reduction technique by optimizing the phase value of pilot tones for the reduction of PAPR in OFDM. However, an exhaustive search and extensive computer simulations have to be conducted to find the optimum values of phases in an IEEE802.11a system.

III. NEW PAPR REDUCTION TECHNIQUE

In this paper, we propose a new technique for PAPR reduction, which can be viewed as an extension of clipping and filtering. In the clipping technique hard limiting is applied to the amplitude of the complex values of the IFFT output. The filtering technique is designed to alleviate out-of-band distortion but cannot correct in-band distortion. Fig. 1 demonstrates the clipping and filtering process [7] [17]. Firstly the data signal X is oversampled by a factor I by placing *N(I-1)* zeros in the center of the vector. This oversampled signal is transformed into the time domain using an IFFT and clipping is performed. Filtering is then performed to remove out of band power and comprises two FFT operations. The in-band discrete components $c_0 \ldots c_{N/2-1}, c_{NI_1-N/2+1} \ldots c_{NI_1-1}$ of the clipped signal are passed unaltered to the input of the second IFFT. The out of band components, $c_{N/2+1} \ldots c_{NI_1-N/2}$, which are located at the center of the vector are set to zero. Although effective at reducing out-of band distortion this technique causes peak regrowth. In order to meet a desired clipping level, recursive clipping and filtering can be employed but this may cause further degradation of the BER.

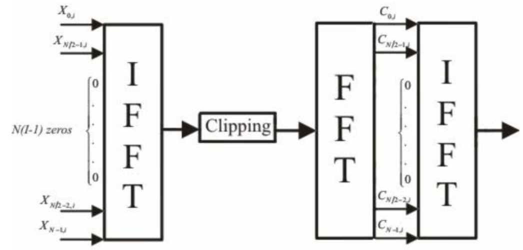

Fig. 1. Clipping and filtering algorithm

We propose a new approach, which also involves clipping but uses the unused carriers and phase information of the pilot tones to reduce the PAPR. It is an iterative process which converges quickly. This algorithm is outlined as follows and is as illustrated in Fig. 2:

I. After an IFFT, the original signal is clipped in the time domain. The clipping can be described by the equation below:

$$c = \begin{cases} |x| e^{j\phi}, |x| \le A \\ A e^{j\phi}, |x| > A \end{cases} \tag{3}$$

where c represents the output of the time domain signal, A is the threshold clipping level and ϕ is the signal phase.

II. The clipped time domain signal c is then converted back into the frequency domain using an FFT. The zeros in m of the outermost unused (free) sub-carriers are replaced with their corresponding clipped values in the frequency domain. The phase of the pilots in the frequency domain can be changed to the corresponding clipped phase in the frequency domain, while the amplitude is restored to a value of 1. All of the remaining values in the frequency domain are restored to their original value before clipping.

III. The new transmit signal is generated after an IFFT. If the PAPR is not below the required threshold, the new values of m of the unused carriers in the frequency domain can be scaled to improve PAPR reduction using a scaling algorithm outlined in IV.

IV. *Scaling algorithm*

Initially the scale factor α_i at the first iteration is set to 1. For α_{i+1} this is increased to β, if this causes a further decrease in PAPR at α_{i+2} the value of β is increased, if not the new scale factor becomes $\beta - \delta$. This process continues until an optimum scale factor is found. Values of β and δ were found empirically.

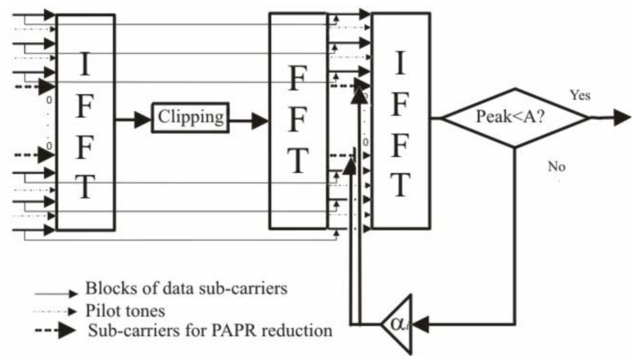

Fig. 2. The proposed technique

The technique proposed above changes the distribution of the signal in the time domain with selected unused carriers and the phase of the pilot tones absorbing the clipping effects. Gatherer et al. [14] propose a similar PAPR reduction technique using the unused tones but do not exploit use of the pilot tones. Also, instead of implementing an effective scaling algorithm which enables the unused tones to absorb much of the clipping effects the signal is continually clipped until a desired threshold is reached. This does not converge easily towards an optimum solution and PAPR often increases after each iteration.

For comparison we compared PAPR reduction using both techniques. Each algorithm uses 3 iterations. The results are shown for an IEEE802.11a system employing 5 unused tones only and no pilot tones for PAPR reduction.

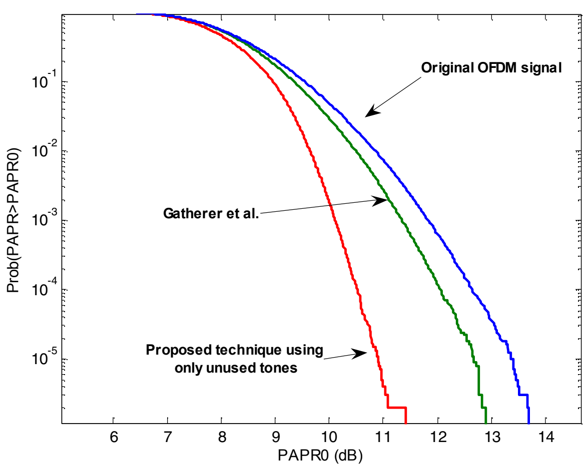

Fig. 3. CCDF showing comparison between Gatherer et al. PAPR reduction technique and proposed new technique using unused tones only in an IEEE802.11a system. 3 itereations are employed for each algorithm.

IV. RESULTS

To verify the proposed PAPR reduction technique, we tested it in the IEEE 802.11a, IEEE802.16e (WiMax) and DVB systems. Here we show the results of a WLAN IEEE 802.11a system. We employed 5 of the unused tones to reduce the PAPR. 16 QAM modulation was employed.

Fig. 4 illustrates the complementary cumulative distribution function (CCDF) plot, in which a decrease in PAPR of up to 4 dB is obtained by employing the new technique in an IEEE802.11a system. On average 4 iterations are required so that an optimum solution can be obtained.

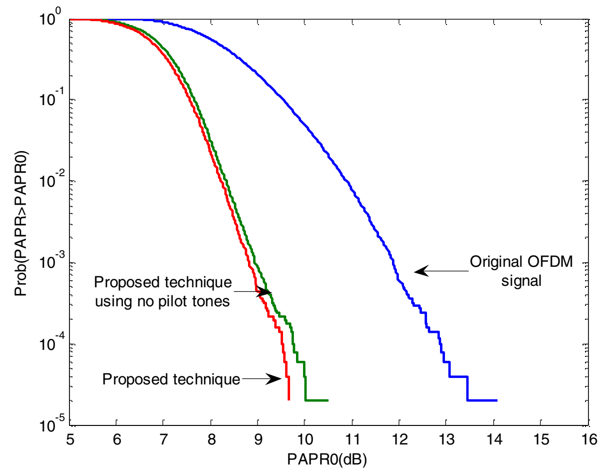

Fig. 4. CCDF for WLAN IEEE802.11a system

Simulations were carried out involving PAPR reduction with use of unused carriers alone and unused carriers with the effect of changing the phase of pilot tones. A PAPR reduction of up to .7dB more can occur when the use of pilot tones is incorporated with unused tones for a WLAN IEEE802.11a system. This has the added benefit of allowing a reduced scaling factor to be employed for the unused tones as an increased amount of the clipping power is absorbed by the phase of pilot tones. This allows the clipping absorption to be spread more evenly.

For comparison, clipping, recursive clipping and filtering and windowing were also implemented in the simulations. For the windowing technique a Gaussian window is employed. A reduced BER is achieved as compared with clipping, recursive clipping and filtering and windowing techniques, as shown in Fig. 5. Unlike a recursive-clipping or windowing technique the BER is not degraded every time when an iteration is performed since no change is being made to the original data-carriers in this new technique. A significant reduction in out-of-band distortion is illustrated in Fig. 6. Compared with clipping and windowing, the proposed technique offers a significant decrease in out-of-band distortion, with only a slight broadening of the original spectrum.

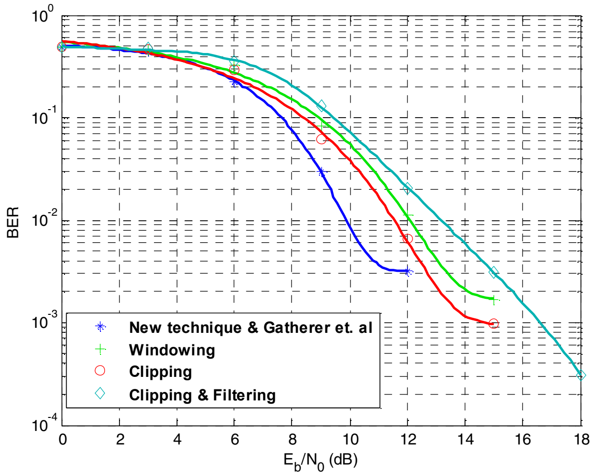

Fig. 5. Bit Error Rate for PAPR reduction of 3.7dB.

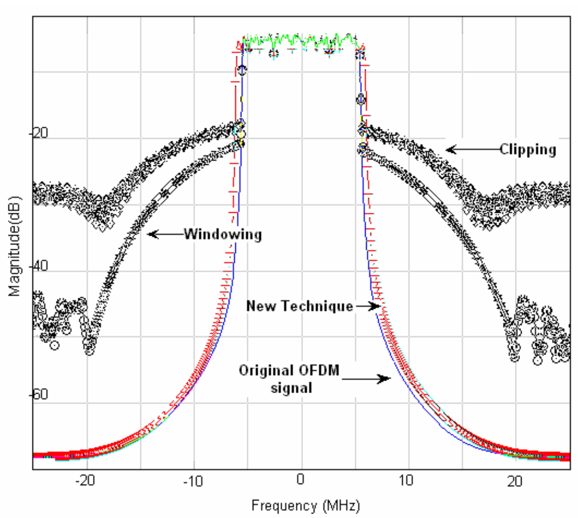

Fig. 6. Output Power Spectra for PAPR reduction of 3.7dB

V. Conclusion

A low complexity PAPR reduction technique has been proposed in this paper. It can significantly reduce high peaks in the OFDM systems but introduces very low out-of-band distortion. It also has a lower BER than clipping and windowing due to no interference to the original data carriers before transmission.

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