PAPR Reduction Technique for OFDM Signals Using Unused Tones with Phase Information

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***Abstract*—A major drawback of Orthogonal Frequency Division Multiplexing (OFDM) is the high Peak-to-Average Power Ratio (PAPR) of the transmit signal which can significantly impact power efficiency and performance. In this paper we present a PAPR reduction technique which exploits the phase of the signal in the frequency domain. When used to extend a previous PAPR reduction technique developed by the authors, a PAPR reduction of up to 5 dB can be obtained with this technique giving extremely low out-of-band distortion as well as lower in-band distortion than windowing and clipping methods. There is also no need for side information to be transmitted to the receiver.**

***Index Terms*—OFDM, PAPR, power amplifier, clipping**

I. Introduction

Orthogonal Frequency Division Multiplexing has many well documented advantages including resistance to multipath fading and high data rates [1-3]. It has therefore been chosen as the modulation standard for IEEE802.11a/g WLAN, Worldwide Interoperability for Microwave Access (WiMax), Digital Audio Broadcasting (DAB) and Digital Video Broadcasting (DVB). However a major drawback of OFDM is the manner in which the phases can align in the frequency domain causing high peaks to result in the time domain.

High peak values cause saturation of the power amplifier and both in-band and out-of-band distortion when limiting effects occur. To prevent such phenomena amplifiers are normally “backed off” by approximately the PAPR. This however severely impacts power amplifier efficiency, making it preferable to reduce the PAPR of the signal before it enters the power amplifier.

Clipping is the most straightforward PAPR reduction technique but can lead to significant out-of-band distortion. In order to alleviate such effects filtering can be applied. However this causes significant peak re-growth. Recursive clipping and filtering can be applied at the price of a severely negative impact on Bit Error Rate (BER) [4]. Windowing is yet another approach [5] with the constraint that the window should be as narrow as possible in the frequency domain, but also not too wide in the time domain otherwise many signal samples are affected and a high BER ensues.

Distortionless techniques such as Tone Reservation [6] can also be employed. Tone Reservation reserves a small number of data tones in the frequency domain to generate an effective cancellation signal for high peaks in the time domain. This can be computationally very burdensome and may cause a significant (up to 20%) reduction in data through-put. It is not very implementable for systems such as IEEE802.11a/g as these have a fairly small number of data carriers and it is impractical to generate an effective cancellation signal in the time domain from only a small number of tones in the frequency domain. Tone Reservation also requires the receiver to know the location of the reserved tones so as to disregard them when decoding the data signal.

Selective Mapping (SLM) [7-9] is implemented by generating a set of sufficiently-different candidate signals from the original data signal. The transmitter selects and submits the candidate signal having the lowest PAPR. Partial Transmit Sequencing (PTS) is a similar technique in which sub-blocks of the original signal are optimally combined at the transmitter to generate a transmitted signal with a low PAPR. Although SLM and PTS are effective at reducing the PAPR, they require the use of side information in order to decode the signal at the receiver.

Previously we developed a PAPR reduction technique which exploits the phase of the pilot tones and a small number of unused tones present in practical OFDM systems [10]. This causes only a slight broadening of the original spectrum and gives an extremely low bit error rate as none of the original data carriers are affected. The low complexity algorithm described in [10] can reduce PAPR by over 3 dB for IEEE802.11a systems in a small number of iterations. In this paper we propose the addition of an extension to this technique which employs the phase of the frequency domain signal. This reduces PAPR reduction by approximately an additional 1dB while maintaining the same out-of band distortion with only a small increase in BER. Overall this new technique offers a lower BER and lower out-of-band distortion than clipping and windowing.

II. Overview of OFDM

In an OFDM system with N sub-carriers, the baseband signal can be written as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j(2\pi k \Delta f t)}, 0 \le t \le T \qquad (1)$$

where X_k is the symbol carried by the k^{th} sub-carrier, Δf is the frequency difference between sub-carriers, and T is the OFDM symbol duration. In the transmitter, the signal or sequence may be generated by the Inverse Fast Fourier Transform (IFFT) of the N-point $\{X_k\}$ sequence, and at the receiver, the Fast Fourier Transform (FFT) is employed to restore the signal. Since the signal which passes through the power amplifier is in the continuous time domain, and may have an instantaneous value significantly higher than the discrete time estimate, it is necessary to oversample the signal by a factor of at least 4 to enable accurate peak detection [11]. For an oversampling factor of I, the input signal to the IFFT is extended by including $N(I-1)$ zeros in the centre of the signal.

The PAPR of the OFDM signal in the time domain can be expressed as

$$PAPR = 10\log_{10}\left(\frac{\max|x(t)|^2}{E\left[|x(t)|^2\right]}\right) \quad (2)$$

where $E[\cdot]$ denotes the expectation operation. As more sub-carriers are added, higher peak values may occur, hence the PAPR increases proportionally with the number of sub-carriers.

III. PAPR Reduction Technique

A. Using Unused Tones and the Phase of Pilot Tones

In practical OFDM-based systems, not all subcarriers are used to transmit the information data, a fact that can be used to reduce the PAPR. Although this results in a slight broadening of the original spectrum, it is possible for the spectrum to remain well inside the spectrum mask defined in the standards [12]. While for optimal channel estimation the pilot tones must be equidistant and equi-powered, and so all pilot tones must have the same magnitude, there is no particular constraint on their phases thereby also leaving them free to exploit for PAPR reduction [13].

Previously we developed a PAPR reduction technique which exploits some unused tones present in practical OFDM systems as well as the phases of the pilot tones in [10]. That technique operates by using a time- and frequency-domain swapping algorithm, which can be described as follows:

I. The time domain signal is hard-clipped.
II. Following an FFT transform, the data carriers and the magnitude of the pilot tones are restored to their original value while a small, pre-selected number of unused tones together with the phases of the pilot tones retain their clipped values.
III. The signal is then converted back into the time domain to check for the presence of peaks. The unused tones may be scaled in the frequency domain to increase their absorption of the clipping effects bringing the time domain peak value down further. An optimal or near-optimal scaling factor can be found after 2 or 3 iterations.

This technique may reduce PAPR by over 3dB with a negligible effect on BER and only a slight broadening of the original spectrum.

B. Using the Phase of Frequency Domain Signal

To further enhance PAPR reduction, we propose to extend the existing PAPR reduction technique by incorporating an idea similar to that proposed in [14][15]. This operates as follows:

I. First apply the technique described in Section A.
II. If the PAPR needs to be further reduced, the time domain signal is hard-clipped again.
III. Following an FFT transform, the amplitudes of the original signal before clipping are restored while the phases of the signal are maintained.
IV. An IFFT is then performed and the signal with a reduced PAPR can be transmitted.

This is different from the method used in [15], where the BER increases rapidly when several iterations are used because the phase distortion becomes significant. Here, by first applying our previous technique, major distortion caused by the clipping is absorbed by the unused carriers and the phase of the pilot tones. In this further iteration, the distortion of the phase is negligible, so that the BER only slightly increases. After this iteration, the PAPR can be further improved by over 1dB, and the out-of-band distortion can be maintained as the same low level as before. Overall the proposed PAPR reduction technique causes much lower BER and out-of-band distortion than clipping and windowing techniques.

IV. Results

To verify the proposed PAPR reduction technique, we tested it in the IEEE 802.11a and the IEEE 802.16e WiMax systems. In the IEEE802.11a system there are 48 data carriers and 4 pilot tones and 12 unused tones while in the WiMax system there are 192 data carriers and 8 pilot tones and 56 unused tones. Oversampling by a factor of 4 is employed in both cases to ensure accurate peak detection and prevention of peak regrowth.

A. PAPR Reduction Capability

Fig. 1 shows the CCDF plot for a 16 QAM IEEE 802.11a signal and Fig. 2 shows the CCDF plot for a 64 QAM IEEE802.11a system, while Fig. 3 shows the results obtained for a 16 QAM WiMax signal. From the results, we can see that PAPR can be reduced by 3-4 dB using our previous technique, while with the additional iteration proposed in this paper, over one more dB can be achieved. Also evident from Figs 1 and 2 is that PAPR reduction capability is not degraded as the order of modulation of the system is increased.

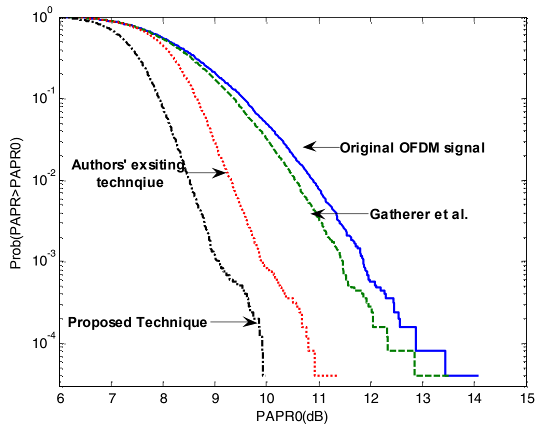

Fig. 1: CCDF plots of a 16QAM 802.11a signal.

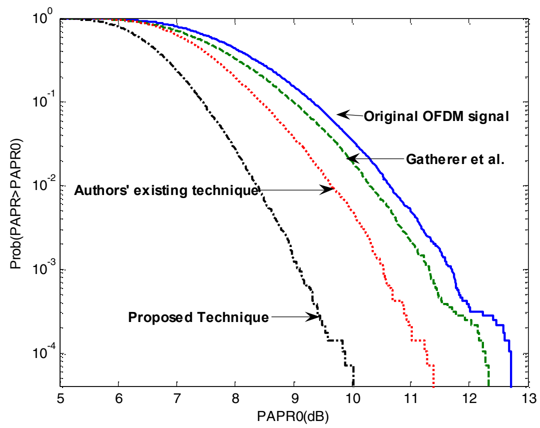

Fig. 2: CCDF plots of a 64QAM 802.11a signal.

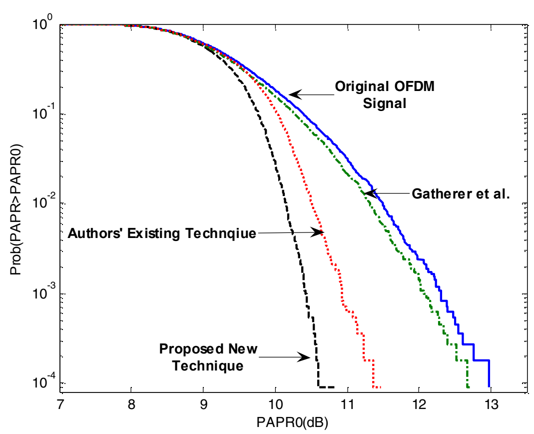

Fig. 3: CCDF plots of a 16QAM 802.16e WiMAX signal.

For comparison, we also included the results from our previous approach and the technique proposed by Gatherer et al. in [16] that also uses unused tones, in which the signal is continually clipped until the appropriate values of unused tones are found. Gatherer's method is slow to converge towards an optimum solution and has limited PAPR reduction as shown in Figs 1-3. For our previous technique and Gatherer's technique an equal number of iterations and unused tones are used in each case. For the results shown 3 iterations are employed although more PAPR reduction can be achieved for a higher number of iterations. In the case of IEEE802.11a 5 tones are reserved for PAPR reduction while 8 are used for WiMax systems.

B. In- band distortion

The proposed PAPR reduction technique causes a lower BER than clipping and windowing techniques since much of the clipping effects have already been absorbed by the unused tones and the phase of pilot tones, as shown in Fig. 4 for the IEEE 802.11a signal and Fig. 6 for the WiMAX signal. For comparison, we also show that, if directly applying the method in [15], i.e., without applying our previous techniques before conducting the clipping and only restoring the amplitudes, which is noted as "extension only" in Fig. 4 and Fig. 6, the BER will become unacceptable after several iterations.

Fig. 5 shows how BER is affected by incorporating the extension to our existing technique. Directly applying the extension only produces a PAPR reduction of 2dB and has almost an identical BER to that produced by utilizing the extension prior to the existing technique. However, if we conduct the extension after applying our existing technique the PAPR then can be reduced by 5 dB but with much lower BER. The reason for this is that using the unused tones and phase of pilot tones has no effect on the original data carriers and causes minimal degradation of BER. Appling the extension immediately after the existing technique results in a significant saving in BER as many of the clipping effects have already been absorbed by the unused tones and phase of pilot tones.

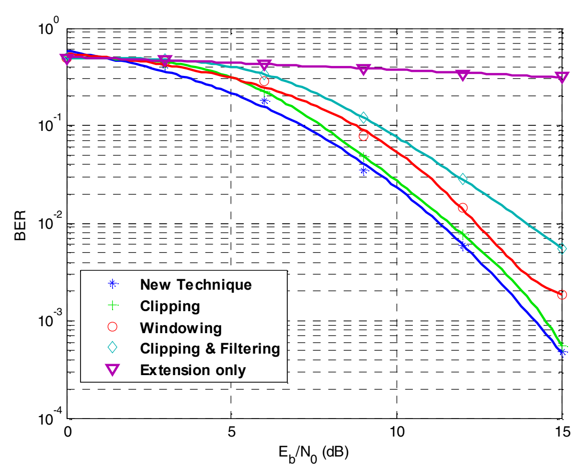

Fig. 4: BER for PAPR reduction of 5 dB for the 16QAM IEEE802.11a system

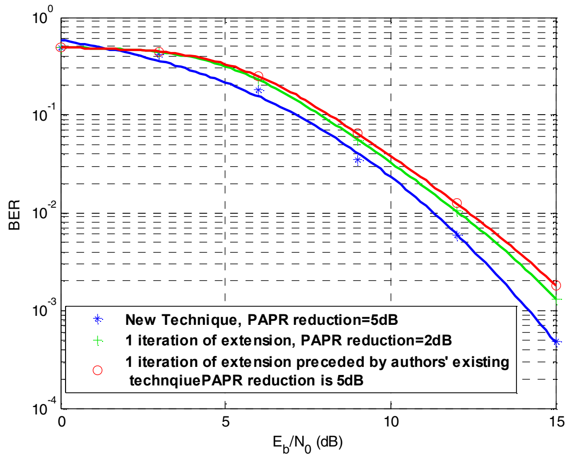

Fig. 5: BER performance comparison with different combinations of the existing technique and the extension for an IEEE802.11a system

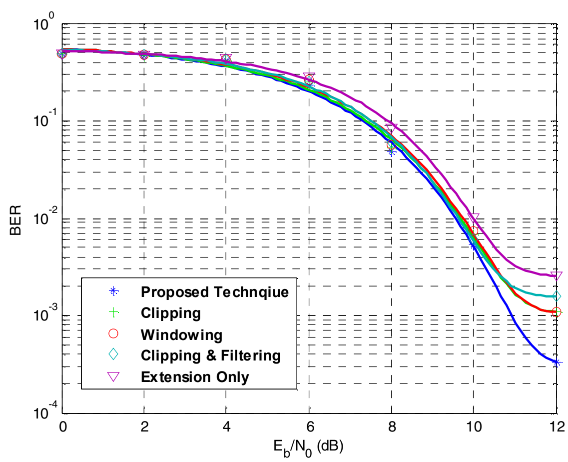

Fig. 6: BER for a WiMax system using 16 QAM. PAPR reduction is 2.5dB for all techniques shown

C. *Out-of-band distortion*

Fig. 7 shows the effect of PAPR reduction on the output power spectrum of an IEEE802.11a system employing 16 QAM where 5 tones are used for PAPR reduction. Fig. 8 shows the resulting power spectrum for a WiMAX system when 4 tones are used for the PAPR reduction. In both cases out-of-band distortion is significantly lower than that of windowing and clipping.

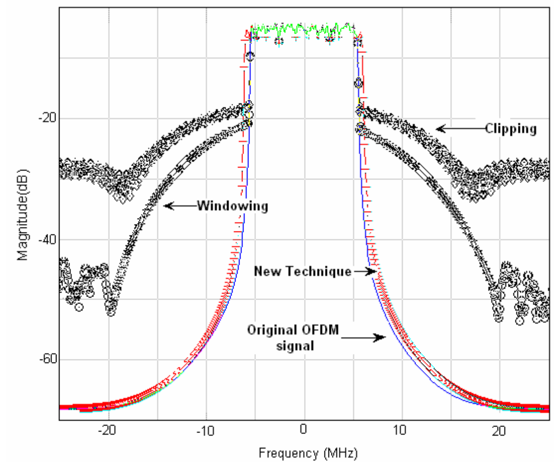

Fig. 7: Output Power Spectrum for PAPR reduction of 4dB for a 16 QAM IEEE802.11a system.

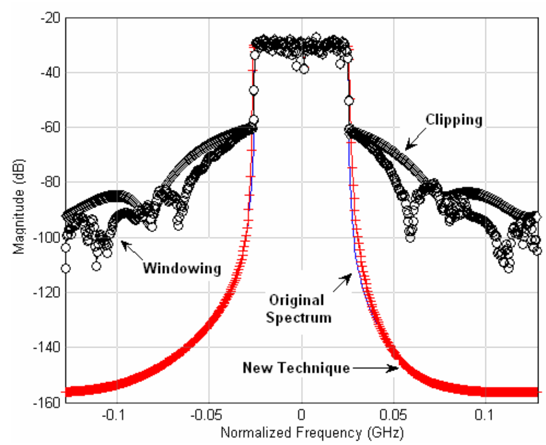

Fig. 8: Output Power Spectrum for PAPR reduction of 2.5dB for 16QAM IEEE802.16e system.

V. Conclusion

We have presented a PAPR reduction technique that exploits the unused tones, the phases of pilot tones, and the phase component of the frequency domain signal for IEEE802.11a and IEEE802.16e systems. This is a low complexity algorithm which can achieve significant PAPR reduction while only causing very limited levels of out-of-band and in-band distortion.

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