Simplified Volterra Series Based Behavioral Modeling of RF Power Amplifiers Using Deviation-Reduction

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***Abstract* — In this paper, a new deviation-reduction approach is proposed to simplify the structure of Volterra series based behavioral models, aimed at significantly reducing the complexity of this kind of model for RF/Microwave power amplifiers. The proposed model reduction method is based on a new format representation of the Volterra series, which is extended from a previously-introduced "Modified Volterra Series" but adapted to a complex baseband formulation in the discrete time-domain. This model can be easily extracted directly from measured time-domain samples of input and output signals of an amplifier by employing simple linear system identification algorithms.**

***Index Terms* — behavioral model, power amplifiers, Volterra series.**

I. INTRODUCTION

RF/Microwave power amplifiers (PAs) play a crucial role in wireless communication systems. Conventional amplifier behavioral models such as AM/AM and AM/PM representations are frequency independent, and can represent with reasonable accuracy the characteristics of various amplifiers driven by narrowband inputs. However, if we attempt to send "wideband" signals, where the bandwidth of the signal is comparable to the inherent bandwidth of the amplifier, we will encounter some frequency dependent behavior in the system. This kind of phenomenon is described under the general heading of "memory effects", and means that the output response of the PA at a given instant depends not only on the input signal at the same time instant but also on the input signal at preceding instants over a short or long term duration. To take account of both nonlinearities and these kinds of memory effects in PA modeling becomes a very important issue in wideband system designs.

The Volterra series has been used by several researchers to describe the relationship between the input and the output of a power amplifier with memory effects [1]. However, high computational complexity makes methods of this kind impractical in some real cases, e.g., modeling a PA with strong nonlinearities and/or with long-term memory effects. This is because the number of coefficients to be estimated in the model increases exponentially with the degree of nonlinearity and with the memory length of the system. To overcome the high complexity of a general Volterra series, a Volterra-like approach, called the "Modified Volterra Series", was proposed in [2-4]. This modified series has the important property that it separates the purely static effects from the memory effects, which are intimately mixed in the classical series. Based on the assumption that the nonlinear memory duration in the device is short enough with respect to the signal period, this series can be truncated to a single-fold integral, which allow modeling not only for weak but also for relatively strong nonlinearities. However, up to date models of this kind have been based on continuous-wave signals, which are not suitable for simulation in system-level simulators in the digital domain, e.g., in MATLAB. Model extraction also requires complicated experimental procedures. In this paper, we further extend this "Modified Volterra Series" to the discrete time domain, and propose a new format of reduced-order behavioral model, which can be directly extracted from time-domain measurement samples of the input and output envelopes of a PA by employing linear system identification algorithms, e.g., least squares (LS). This new model can be easily embedded in system level simulation tools.

II. MODIFIED VOLTERRA SERIES

The Volterra series is a combination of linear convolution and nonlinear power series. A finite-order nonlinear system with finite memory can be represented by a truncated Volterra series in the discrete time domain as

$$y(n) = \sum_{p=1}^{P} y_p(n) \qquad (1)$$

where

$$y_p(n) = \sum_{i_1=0}^{N-1} \cdots \sum_{i_p=0}^{N-1} h_p(i_1, \cdots, i_p) \prod_{j=1}^{p} x(n-i_j) \qquad (2)$$

where $x(n)$ and $y(n)$ is the input and the output, respectively, and $h_p(i_1, \cdots, i_p)$ is called the "*Volterra kernel*". P is the highest order of nonlinearity and N represents the maximum memory length of the system.

Since it is very difficult to identify Volterra kernels of order greater than five in a practical way, this formulation of the Volterra series can only be used for modeling weakly nonlinear systems. In order to overcome this limitation, a simplified Volterra-like approach was proposed in [2-4], based on introducing the dynamic deviation function $e(n,i)$,

$$e(n,i) = x(n-i) - x(n) \qquad (3)$$

which represents the deviation of the delayed input signal $x(n-i)$ with respect to the current input $x(n)$. By substituting (3) in (2), it is immediately apparent that the output signal

$y(n)$ in (1) can be expressed through the following dynamic deviation based Volterra-like series:

$$y(n) = y_s(n) + y_d(n) \tag{4}$$

where $y_s(n)$ represents the static characteristic of the system, and can be expressed as a power series of the current input signal $x(n)$:

$$y_s(n) = \sum_{p=1}^{P} a_p x^p(n) \tag{5}$$

while $y_d(n)$ is a purely-dynamic multi-dimensional convolution with respect to the dynamic deviation and is controlled by the current input signal $x(n)$:

$$\begin{aligned} y_d(n) = & \sum_{p=1}^{P}\sum_{r=1}^{p} x^{p-r}(n) \\ & \cdot \sum_{i_1=0}^{N-1}\cdots\sum_{i_r=0}^{N-1} g_{p_r}(i_1,\cdots,i_r)\prod_{j=1}^{r} e(n,i_j) \end{aligned} \tag{6}$$

In many applications, it is possible to truncate the system memory to a finite, relatively short time interval centered on the current instant at which the output is evaluated. If such a memory time is "short" enough, then the dynamic deviation $e(n,i)$ assumes small values for that memory duration. Therefore, $y_d(n)$ can be linearized with respect to $e(n,i)$, leading to

$$y(n) = \sum_{p=1}^{P} a_p(n) x^p(n) + \sum_{p=1}^{P} x^{p-1}(n) \sum_{i=0}^{N-1} g_{p_1}(i) e(n,i) \tag{7}$$

in which only the first-order kernel of the dynamic deviation-based Volterra series needs to be identified in order to characterize the nonlinear system with memory [2-4]. In this model, the static nonlinearities and the dynamic deviation parts can be estimated separately. The static nonlinear parts are generally extracted under large-signal conditions in the frequency domain, while the characterization of $g_{p_1}(i)$ is based on small-signal operating conditions. This model extraction involves complicated experimental procedures [3].

However, if we re-substitute (3) in (7), then after some re-arrangement it can be shown that:

$$y(n) = \sum_{p=1}^{P} h_p(n) x^p(n) + \sum_{p=1}^{P} x^{p-1}(n) \sum_{i=1}^{N-1} h_{p_1}(i) x(n-i) \tag{8}$$

Compared to (7), the advantage of the new format in (8) is that the output of the model is still linear with respect to the coefficients, a property also possessed by the classic Volterra model in (1). This property of linearity allows us to employ linear system identification algorithms to extract the nonlinear Volterra model in a direct way.

Following a similar format to that of (8), we could rewrite (1) in a new representation as:

$$\begin{aligned} y(n) = & \sum_{p=1}^{P} h_p(n) x^p(n) \\ & + \sum_{p=1}^{P}\{\sum_{r=1}^{p}[x^{p-r}(n)\sum_{i_1=1}^{N-1}\cdots\sum_{i_r=i_{r-1}}^{N-1} \\ & \cdot h_{p_r}(i_1,\cdots i_r)\prod_{j=1}^{r} x(n-i_j)]\} \end{aligned} \tag{9}$$

Compared to (2), we can see that the sequence of the input product items in the input vector has been re-organized in (9). Employing this representation, we could easily derive an effective model reduction approach as discussed later.

III. Deviation Reduction

In the new representation of the Volterra series in (9), r is directly related to the possible order of product terms of the delayed inputs in the input vector. For instance, $r=1$ means only one delayed input is included in the product, i.e., $x^{p-1}(n)x(n-i_j)$. It is also easy to see that the first-order deviation based Volterra model in (8) is the truncation of the general model (9) by limiting $r=1$.

As discussed in [5], a first-order truncation of a dynamic Volterra series expansion permits accurate modeling of highly nonlinear systems, but its effectiveness tends to be limited to those systems where the nonlinear memory duration is sufficiently small compared to the inverse of the bandwidth of the envelope signal. Unfortunately, it is found in practice that many solid state amplifiers exhibit effective nonlinear memory duration much longer than the inverse of their operating bandwidth - especially due to thermal and bias circuit modulation effects. In order to handle long-term memory effects or high order dynamics, more terms need be added in (8) to improve the accuracy of the model.

Fortunately, in practice, memory effects in amplifiers tend to decline with time, which means that longer time-delayed inputs have less effect on the output signal. Meanwhile, the effects of nonlinear dynamics also fade with increasing order [3]. This means that it is reasonable to limit r to a small value, e.g., $1 \le r \le M$, in order to reduce the complexity of the model but still keep good accuracy. We describe this model reduction criterion as "Deviation Reduction" since r represents the order of the dynamic deviation in the model. For example, by setting $1 \le r \le 2$, the dynamic parts in (9) are truncated to second-order, so that the truncated Volterra model becomes:

$$\begin{aligned} y(n) = & \sum_{p=1}^{P} h_p(n) x^p(n) \\ & + \sum_{p=1}^{P}[x^{p-1}(n)\sum_{i=1}^{N-1} h_{p_1}(i) x(n-i)] \\ & + \sum_{p=2}^{P}[x^{p-2}(n)\sum_{i_1=1}^{N-1}\sum_{i_2=i_1}^{N-1} \\ & \cdot h_{p_2}(i_1,i_2) x(n-i_1) x(n-i_2)] \end{aligned} \tag{10}$$

The decision as to how to select the truncation order depends on the practical characteristics of power amplifiers and the model fidelity required.

IV. Low-pass Equivalent Volterra Model

In communication system analysis and design, most system level simulators use baseband complex envelope signals to evaluate the system performance since modulation techniques are normally employed in modern communication systems. As a result, it is necessary to develop a complex baseband or low-pass equivalent representation for the Volterra models in order to handle carrier-modulated signals.

Consider $x(t) = \mathrm{Re}[\tilde{x}(t)e^{j\omega_0 t}]$ and $y(t) = \mathrm{Re}[\tilde{y}(t)e^{j\omega_0 t}]$ as the input and output signals of a power amplifier, where ω_0 is carrier frequency and $\tilde{x}(t)$ and $\tilde{y}(t)$ represents the complex-valued envelope of the input and output signals, respectively. Following the same procedure as used to derive (9), the new low-pass equivalent Volterra model can be written in the discrete time-domain as

$$\tilde{y}(n) = \sum_{i=0}^{r} \tilde{y}_i(n) \tag{11}$$

where

$$\tilde{y}_0(n) = \sum_{k=0}^{\frac{P-1}{2}} \tilde{h}_0(k)\left|\tilde{x}(n)\right|^{2k} \tilde{x}(n) \tag{12}$$

$$\begin{aligned}\tilde{y}_1(n) = &\sum_{i=1}^{N-1} \tilde{h}_1(i)\tilde{x}(n-i) \\ &+\sum_{k=1}^{\frac{P-1}{2}} [\left|\tilde{x}(n)\right|^{2k} \sum_{i=1}^{N-1} \tilde{h}_{k_{11}}(i)\tilde{x}(n-i)] \\ &+\sum_{k=1}^{\frac{P-1}{2}} [\left|\tilde{x}(n)\right|^{2(k-1)} \tilde{x}^2(n)\sum_{i=1}^{N-1} \tilde{h}_{k_{12}}(i)\tilde{x}^*(n-i)]\end{aligned} \tag{13}$$

$$\begin{aligned}\tilde{y}_2(n) = &\sum_{k=0}^{\frac{P-3}{2}} [\left|\tilde{x}(n)\right|^{2k} \tilde{x}^*(n)\sum_{i_1=1}^{N-1}\sum_{i_2=i_1}^{N-1} \tilde{h}_{k_{21}}(i_1,i_2)\tilde{x}(n-i_1)\tilde{x}(n-i_2)] \\ &+\sum_{k=0}^{\frac{P-3}{2}} [\left|\tilde{x}(n)\right|^{2k} \tilde{x}(n)\sum_{i_1=1}^{N-1}\sum_{i_2=1}^{N-1} \tilde{h}_{k_{22}}(i_1,i_2)\tilde{x}(n-i_1)\tilde{x}^*(n-i_2)] \\ &+\sum_{k=1}^{\frac{P-3}{2}} [\left|\tilde{x}(n)\right|^{2(k-1)} \tilde{x}^3(n)\sum_{i_1=1}^{N-1}\sum_{i_2=i_1}^{N-1} \tilde{h}_{k_{23}}(i_1,i_2)\tilde{x}^*(n-i_1)\tilde{x}^*(n-i_2)]\end{aligned} \tag{14}$$

$$\vdots$$

Here, $(\bullet)^*$ represents conjugate transpose while $|x|$ returns the complex magnitude of x. $\tilde{y}_0(n)$ is the static part, while $\tilde{y}_1(n)$ and $\tilde{y}_2(n)$ represents the first-order and the second-order dynamic term, respectively. The high order terms can be derived in the same way. In real applications, in order to reduce the model complexity, only limited terms are kept in (11) by employing the Deviation-Reduction procedure, i.e., limiting r to a small value.

V. Model Extraction

A key input requirement for power amplifier behavioral modeling is that the ensemble of input waveforms contains most frequencies and amplitudes of interests, i.e., covers the entire bandwidth and dynamic range of the PA under study. In other words, the system must be tested exhaustively in order to observe all possible nonlinear interactions. This clearly implies that the system cannot be tested separately for different frequency bands and amplitude ranges, e.g., using traditional single-tone power/frequency sweep tests, since the superposition principle does not hold for nonlinear amplifiers and nonlinear interactions must be observed otherwise they cannot be modeled.

Recently, a time-domain stimulus-response measurement solution has been proposed by Agilent Technologies [6], which uses complex envelope waveforms as the input excitation. The modulated data files are first created at baseband and downloaded to the arbitrary waveform generator that feeds the complex I and Q signals to the IQ-modulator in the signal generator. The signal generator produces the test signal to the device under test (DUT). The output of the DUT is then down-converted and sampled by the Vector Signal Analyzer (VSA). The sampled input and output data are finally used to extract behavioral models for the power amplifier.

As mentioned earlier, the output of the Volterra model is linear with respect to its coefficients. In other words, we can say that the coefficients appearing in (9) are a generalization of the impulse response coefficients $h(\cdots)$ defining a linear model. Consequently, one possible approach to the problem of Volterra model parameter estimation is to treat it as a large but standard regression problem. In particular, we could form a single large parameter vector $\boldsymbol{\theta}$ containing all of the unknown coefficients $h(\cdots)$ and define the matrix $\mathbf{X}$ including all of the product terms $x(n-i_1)\cdots x(n-i_N)$ appearing in the model for $n = N+1,\cdots,L$, where L is the total length of the available data record. If we assume the presence of an unmodeled error $\mathbf{e} = [e(N+1),\cdots,e(L)]^T$, the Volterra model can be written as:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\theta} + \mathbf{e} \tag{15}$$

where $\mathbf{y} = [y(N+1),\cdots,y(L)]^T$. A popular solution to this problem is the least squares (LS) method, in which $\boldsymbol{\theta}$ is estimated as that value $\hat{\boldsymbol{\theta}}$ minimizing the model error criterion:

$$J(\boldsymbol{\theta}) = \sum_{n=N+1}^{L} e^2(n) = \mathbf{e}^H\mathbf{e} \tag{16}$$

A standard result states that the estimate minimizing this criterion is:

$$\hat{\boldsymbol{\theta}} = (\mathbf{X}^H \mathbf{X})^{-1} \mathbf{X}^H \mathbf{y} \qquad (17)$$

This result has the advantage of notational simplicity and general applicability. Other linear adaptive techniques, including the RLS (recursive least squares) and the LMS (least mean squares) algorithms, also can be employed to estimate the model parameters here.

VI. Experimental Results

In order to validate the proposed behavioral model, we test a HBT power amplifier, which has noticeable memory. This power amplifier is operated at 2.14 GHz and excited by downlink 3GPP W-CDMA signals of 3.84 Mcps chip rate and peak-to-average power ratio equal to 8.2 dB @ 0.01% probability on CCDF. The test bench setup uses the ADS-ESG-VSA connected solution [6]. Around 12,000 sampling data points are captured from the input and output envelope signals of the PA.

In this test, we truncated the Volterra model to 5th-order, and set the memory length to 4. For comparison, we first truncated the model to first-order, then extended to higher order, in other words, increased r from 1 to 4. The NMSEs (normalized mean square errors) were calculated to evaluate the model fidelity in the time domain, shown in Table I.

TABLE I
MODEL PERFORMANCE IN THE TIME DOMAIN

Order of Deviation	1	2	3	4
NMSE (dB)	-32.3	-38.2	-38.8	-39.1
Number of Coefficients	18	48	178	244

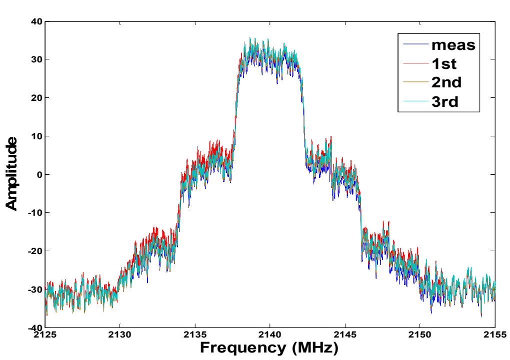

Fig. 1. Spectra of the output in the frequency domain

From Table I, we can see that the performance of the first-order model is quite poor. The 4th-order model achieves high accuracy but the number of coefficients increases significantly. The frequency-domain spectra of the outputs are shown in Fig. 1. From these results, we may draw the conclusion that low-order nonlinear dynamics dominate nonlinear distortion caused by the PA's memory effects. Therefore it is reasonable to remove high-order dynamics or high-order nonlinear memory effects in the model in order to reduce the model complexity since their effects quickly fade with increasing order. While the model structure becomes simpler after model reduction, we can increase the maximum order of nonlinearity, i.e., P, to cover higher order nonlinear effects, so that strongly nonlinear systems can be modeled using this model. In addition, we may increase the memory length N to characterize long-term linear and low-order nonlinear memory effects.

VII. Conclusions

A new format of Volterra series has been introduced in this paper. Based on this new representation, we propose a "Deviation-Reduction" method to simplify the model structure, which may significantly reduce the complexity of Volterra series based behavioral models of RF power amplifiers. Using this model reduction approach, we can effectively trade off between the model simplicity and the model fidelity, which make the application of the Volterra model more flexible in practical applications. Furthermore, a model of this kind can be easily extracted from time-domain measurements and quickly implemented in system level simulators.

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