Gaussian Pulse Based Tone Reservation for Reducing PAPR of OFDM Signals

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***Abstract* – One of the major drawbacks of OFDM is high Peak-to-Average Power Ratio (PAPR) which can result in poor power efficiency and serious distortion in the transmitter amplifier. Tone Reservation (TR) is a technique designed to combat this problem by reserving a number of carriers (tones) in the frequency domain to generate a cancellation signal in the time domain to remove high peaks. However TR can have a high associated computational cost due to the difficulties in finding an effective cancellation signal in the time domain by using only a few tones in the frequency domain. In this paper, we propose a novel approach to overcoming this problem by creating a Gaussian pulse as the cancellation signal from only a small number of reserved tones. This facilitates a simple and effective algorithm for reducing peak values while minimizing the occurrence of secondary peaks, the latter being a key factor in contributing to the high computational complexity of tone reservation algorithms.**

***Index Terms* — OFDM, peak-to-average-power-ratio (PAPR), tone reservation, DVB, WiMax.**

I. Introduction

Orthogonal Frequency-Division Multiplexing (OFDM) offers many well-documented advantages for multi-carrier transmission at high data rates, including high spectral efficiency, simple implementation using the Fast Fourier Transform (FFT), immunity to multi-path fading etc. [1-4]. However, one of major drawbacks of OFDM is the high peak-to-average power ratio (PAPR) of the transmitted signal. This occurs due to the large number of independent sub-carriers with random phase that are added together at the modulator.

The high peaks occur very rarely, but the power amplifier (PA) may be overdriven deep into saturation on these rare occurrences since we might normally wish to operate the PA under reasonably strong drive conditions to maintain high power efficiency. During the overdrive event the distortion that is generated results in very high instantaneous spectral re-growth which can cause serious adjacent channel interference. To prevent these effects, the amplifier must be backed-off from its maximum output power by approximately the PAPR of the input data, which results in very low power efficiency. For example, the typical power efficiency of a class AB PA is only around 18% in an IEEE 802.11a system [5]. It is therefore preferable if possible to reduce the PAPR of the signal to avoid the use of back-off.

Several techniques have been developed to address the PAPR problem. The simplest is clipping, but it causes in-band and out-of-band distortion. Filtering can be used to alleviate out-of-band distortion but results in peak re-growth. Repeated clipping and filtering can lead to serious degradation in bit-error-rate (BER) [6]. The use of a windowing technique involves multiplying large signal peaks by a non-rectangular window such as a Gaussian pulse to minimize the out-of-band interference. Ideally the window should be as narrow-band as possible but it should not have too long an extension in the time domain, otherwise more signal samples are affected, which would result in an increase in BER [7].

Selective Mapping (SLM) is another approach, which is implemented by generating a set of sufficiently different candidate signals from the original data signal. The transmitter selects and submits the candidate signal which has the lowest PAPR. Partial Transmit Sequencing (PTS) is a similar technique in which sub-blocks of the original signal are optimally combined at the transmitter for generating a transmitted signal with a low PAPR. Although SLM and PTS are effective at reducing the PAPR, they require the use of side information to the receiver in order to decode the signal [8-10].

The tone reservation (TR) algorithm was developed by Tellado [11], whereby a small number of sub-carriers (tones) are reserved to create a signal which can cancel the high peaks in the information-carrying signals at the transmitter. This approach can reduce the PAPR of the OFDM signals without introducing any additional distortions to the information data and does not require side information. However, TR can have a high computational cost due to the difficulties of finding an effective cancellation signal in the time domain from only a small number of reserved tones in the frequency domain. In this paper, we propose a novel approach for overcoming this difficulty by creating a Gaussian-pulse-like cancellation signal which facilitates a simple procedure for reducing peak values, while minimizing the occurrence of secondary peaks.

II. Overview of Tone Reservation

In an OFDM system with N sub-carriers, the transmitted base band signal can be written as

$$x(t)=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}X_k e^{j(2\pi k\Delta f t)},0\le t\le T \tag{1}$$

where X_k is the symbol carried by the kth sub-carrier, Δf is the frequency difference between sub-carriers, and T is the OFDM

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symbol duration. In the transmitter, the transmitted signal or sequence may be generated by the Inverse Fast Fourier Transform (IFFT) of the N-point $\{X_k\}$ sequence, and at the receiver, the Fast Fourier Transform (FFT) is employed to restore the signal, e.g.,

$$x_n = IFFT(X_k) \text{ and } X_k = FFT(x_n) \tag{2}$$

Because the continuous-time peak power can be significantly higher than in the discrete-time domain, it is necessary to oversample the signal by a factor of at least four [12]. For an oversampling factor of I, the input signal to the IFFT is extended by including $N(I$-1) zeros in the center of the signal.

The PAPR of the transmitted signal can be expressed as:

$$PAPR(x) = 10\log_{10}\left(\frac{\max|x(n)|^2}{E\left[|x(n)|^2\right]}\right) \tag{3}$$

where $|x(\text{n})|$ returns the magnitude of $x(\text{n})$, and $E[\cdot]$ denotes the expectation operation. As more sub-carriers are added, higher peak values may occur, hence the PAPR normally increases proportionally with the number of sub-carriers.

In Tone Reservation, a small number of sub-channels (tones), which do not carry any information data, are reserved for peak cancellation. This restricts the data-bearing vector X, and the reserved tone vector C to lie in disjoint frequency subspaces, i.e., they cannot both be nonzero at a given tone, which requires that

$$X_K + C_K = \begin{cases} C_K, K \in L \\ X_K, K \in L^c \end{cases} \tag{4}$$

The subset of reserved tones can be denoted by $L=\{i_0,\ldots,i_L\}$ with $L << N$, where N represents the set of all tones in the multi-carrier symbol. L^C is the complement of L in N and represents information carriers. The addition of these reserved tones c to a data-bearing signal x produces a new composite signal

$$\bar{x}[n] = x[n] + c[n] = IFFT(X_K + C_K) \tag{5}$$

Since symbol demodulation is performed in the frequency domain on a tone-by-tone basis, the reserved sub-channels can be discarded at the receiver, and only the data-bearing sub-channels are used to determine the transmitted bit stream. The new PAPR becomes

$$PAPR(\bar{x}) = 10\log_{10}\left(\frac{\max|x(n)+c(n)|^2}{E\left[|x(n)|^2\right]}\right) \tag{6}$$

where we can see that the PAPR can be reduced by optimizing $c(\text{n})$ so that $max|x(\text{n})+c(\text{n})|^2$ can be smaller than $max|x(\text{n})|^2$.

III. Generation of Gaussian Pulses for Reserved Tones

In the tone reservation approach, a small number of sub-carriers (tones) are reserved to create a signal which cancels the high peaks of information-carrying signals in the transmitter. This cancellation signal must be generated in the frequency domain using the minimum number of tones to maximize data throughput, however it is also preferable to have a narrow time domain signal to prevent the generation of secondary peaks. In other words, the PAPR reduction approach in tone reservation is a constrained signal-design problem: a signal must be designed in the frequency domain, but its effect is evaluated in the time domain. In current tone reservation approaches, the cancellation signal is mainly generated from either trial and error processes or involves computationally complex optimization procedures [11][13].

In this paper, we propose a simple algorithm, in which a Gaussian window-like signal is employed in the frequency domain to form the canceling pulse in the time domain. Since Gaussian pulses can be optimized in both the time domain and the frequency domain [14]. By selecting a small number of tones in the frequency domain, we can generate a narrow pulse in the time domain, as shown in Fig 1.

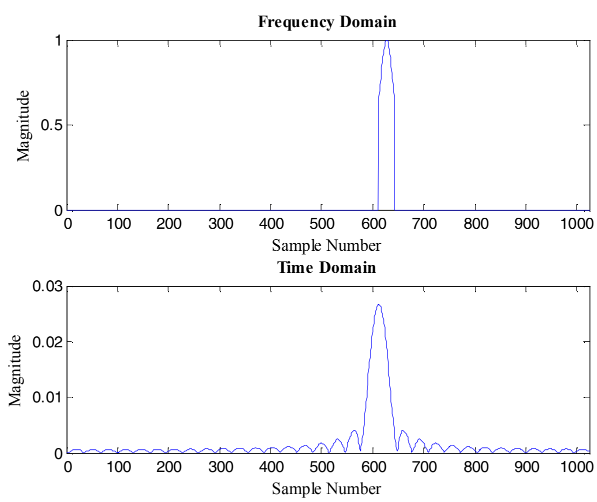

Fig. 1 Cancellation signal formed from Gaussian pulse in the time and the frequency domain

The coefficients of the Gaussian window, G, can be calculated from the equation:

$$G_{[m+1]} = e^{-\frac{1}{2}\left(\alpha\frac{m-\frac{L}{2}}{L/2}\right)^2} \quad \text{where } 0 < m < L-1 \tag{7}$$

where α represents the reciprocal of the standard deviation, and the width of the window is inversely related to α. These values represent the amplitude of the Gaussian window and the phase has a value of zero. The value of L represents the number of tones reserved to generate the cancellation signal. Typically, a value of 16 or 32 provides sufficiently narrow signals in the time domain to avoid the occurrence of secondary peaks. Unlike previously proposed methods, the generation of this cancellation signal does not require complicated peak searching or optimization procedures [11][13]. This single peak signal is easily optimized both in the time- and the frequency-domains. Just one time IFFT operation is required, and very few tones are needed. For example, in a WiMax system with over-sampling by a factor of four, a selection of 16 tones only occupies 1.5% of the available bandwidth.

IV. Algorithm for Reducing PAPR

Once an efficient cancellation signal is obtained, a fast conversion algorithm can be applied to the OFDM system to cancel the high peaks so that the transmitted signal does not exceed the required threshold A. The algorithm employed is implemented as follows:

I. A pre-defined cancellation signal is generated by using the Gaussian pulses described in Section III. This cancellation signal only has non-zero values in the reserved tone locations in the frequency domain and has one sharp peak in the time domain.

II. We first check if there are peaks exceeding the required threshold A in the information-carrying signal. If there are, the magnitude of the peaks and their corresponding location are detected.

III. For each peak detected in the information data, the peak of the pre-defined cancellation signal is circularly shifted to the peak location and scaled by the value of the difference between the peak and the threshold so that the power of the peak tone can be reduced to the desired target level. All of the appropriately scaled and phase shifted cancellation signals are then subtracted from the original information signal.

IV. After the peak cancellation, the composite signal will be detected again since some secondary peaks may appear during the previous peak-canceling operation. The process is continued until all the peaks are below the required threshold or until a maximum number of iterations are exceeded, as shown in Fig. 2.

V. The new time domain signal with reduced PAPR at iteration i can be expressed as:

$$\overline{x}^{i+1}[n] = \overline{x}^{i}[n] - \mu_i p\left[\left(n-n_i\right)_N\right] \tag{8}$$

The cancellation signal c_i at iteration i is represented by $\mu_i p[(n-n_i)_N]$ where μ_i represents the scaling factor and phase rotation applied for peak cancellation and $p[(n-n_i)_N]$ denotes the necessary circular shift of the cancellation signal so that the peaks are cancelled.

Since the Gaussian-pulse based peak cancellation signal has a very sharp shape in the time domain, the occurrence of secondary peaks is minimized. Typically only a single iteration is required to remove each peak detected in the original time-domain signal. In this approach, when the pulse cancellation signal is circularly shifted, scaled and phase-rotated in the time-domain, the values of the frequency-domain signal only change at the reserved tone locations but remain unchanged at the other tones. The data vector X is not affected by the peak canceling operations, so that it does not need any side information or any receiver operation.

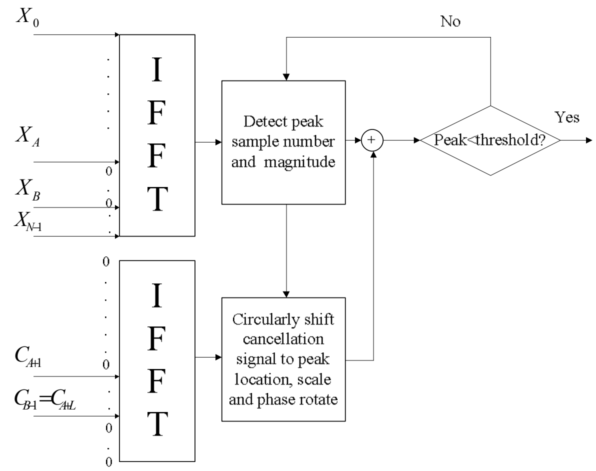

Fig. 2 Representation of procedure for peak cancellation

V. Results

To verify the approach proposed, we first simulated it in a WiMax system, which contains 256 data carriers with 16-QAM modulation and a bandwidth of 16 MHz. The cancellation signal was generated by using 16 tones. A sample of the time domain signal is shown in Fig. 3, where we can clearly see that the high peaks were effectively removed after three iterations. The PAPR was reduced by over 4 dB, as shown in the complementary cumulative distribution function (CCDF) plot in Fig. 4.

The Digital Video Broadcasting (DVB) standard is also referred to as the “2K-mode”, in which 2048 data carriers are employed and with the bandwidth of 8 MHz and the modulation technique used is 64 QAM. We also used 16 reserved tones to cancel the high peaks in this system, which occupies 0.8% of the total carrier spectrum. The CCDF plot is shown in Fig. 5, where a 3 dB PAPR reduction was achieved.

VI. Conclusion

In this paper, we have proposed an effective solution for generating peak cancellation signals in which only one IFFT operation and simple iterative operations are needed. This dramatically reduces the computational complexity of conventional tone reservation operations. Significant PAPR reduction can be achieved by using only a small number of tones in the frequency domain. This technique can be applied to any communication systems where OFDM is employed, such as wireless LAN or WiMax.

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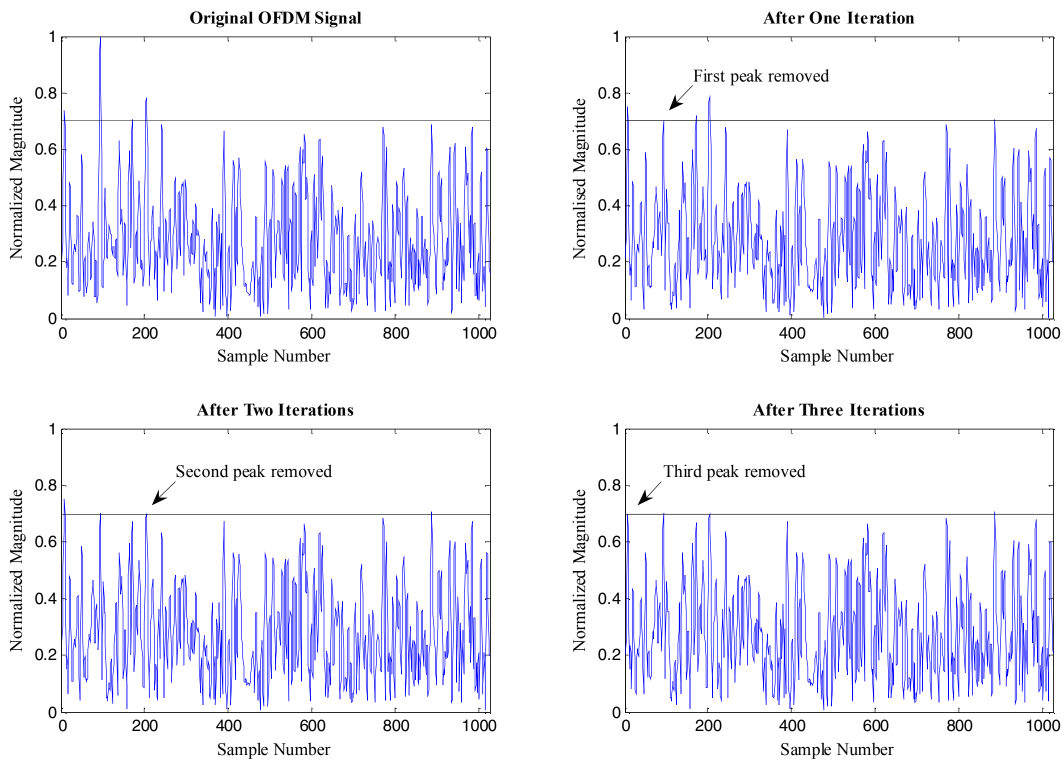

Fig. 3 Time domain sample of a WiMax signal with Tone Reservation

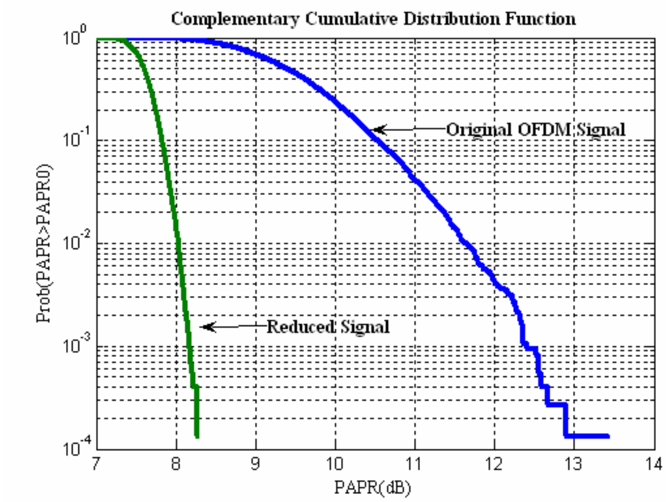

Fig. 4 CCDF plot for a WiMax signal

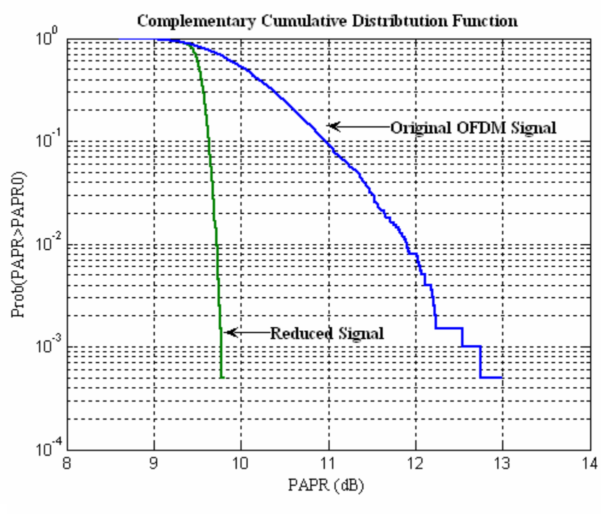

Fig. 5 CCDF plot for a DVB signal